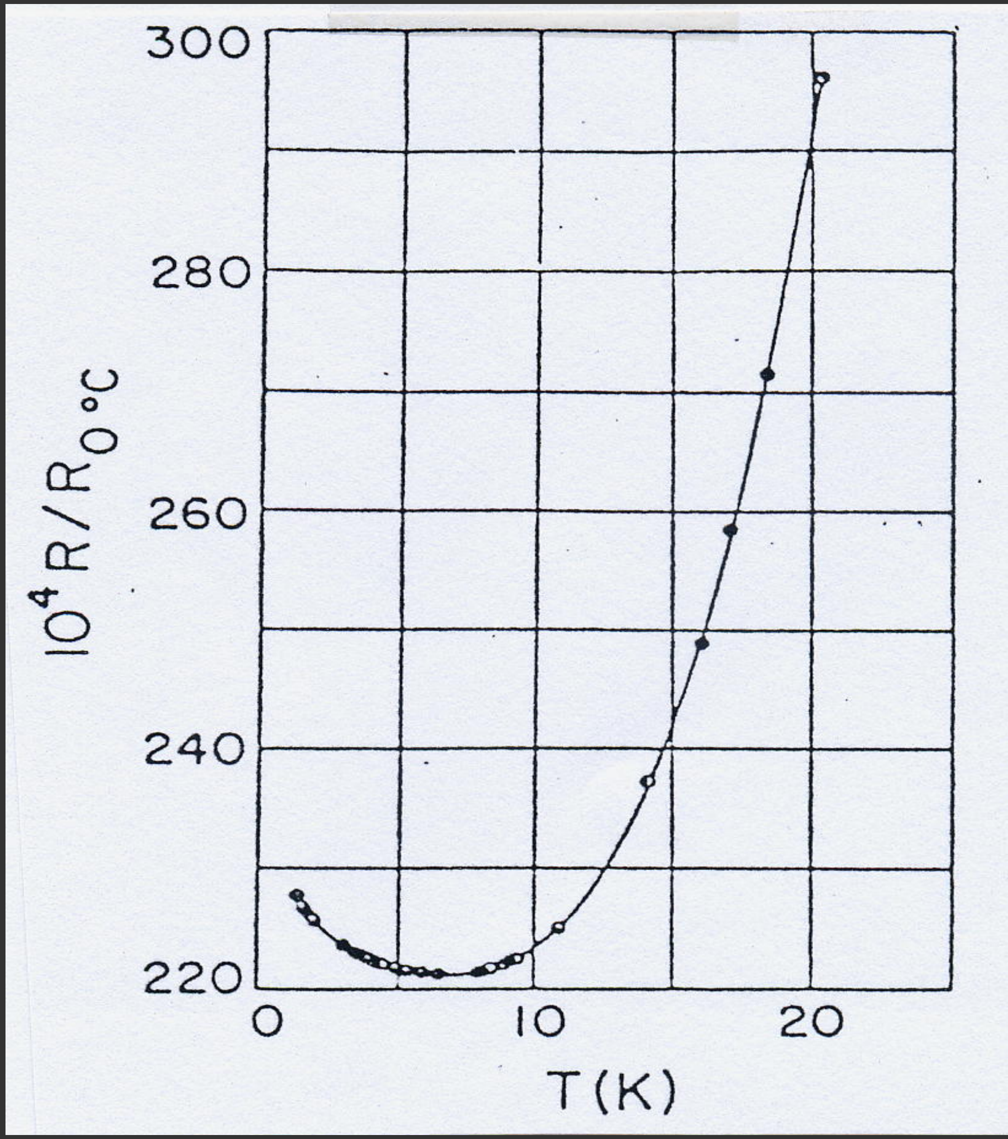
The Kondo effect

- the resistivity minimum
- the Anderson model
- the Kondo solution
- the Kondo problem
- Kondo effect in low dimensional systems

References:

- "*The Kondo Problem to Heavy Fermions*" *by* A. C. Hewson (1995).
- "Kondo resonance phenomena in low-dimensional correlated electron systems" Guang-Ming Zhang and Lu Yu, Physics 36, 434 (2007)

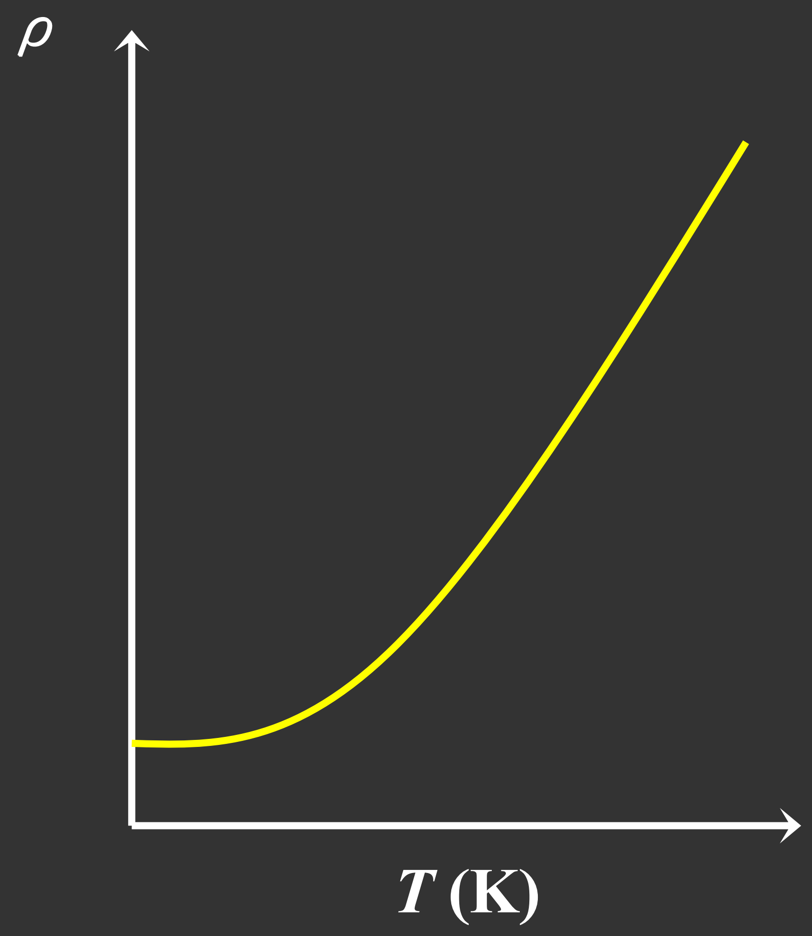
Resistivity minimum puzzle



- In 1930's, experiments show that in many metals ρ has a minimum value at low temperature
- Shown on the left is the original data on Au from de Hass and van den Berg *et al*. from Leiden (1934).
- For a long time no one knows what causes such a non-monotonic resistive behavior.

- 1953: A.H. Wilson "the cause of the minimum is entirely obscure and constitutes a most striking departure from Mathiessens' rule, according to which the ideal and residual resistances are additive – some new physical principle seems to be involved"

Resistivity of metals at low *T*

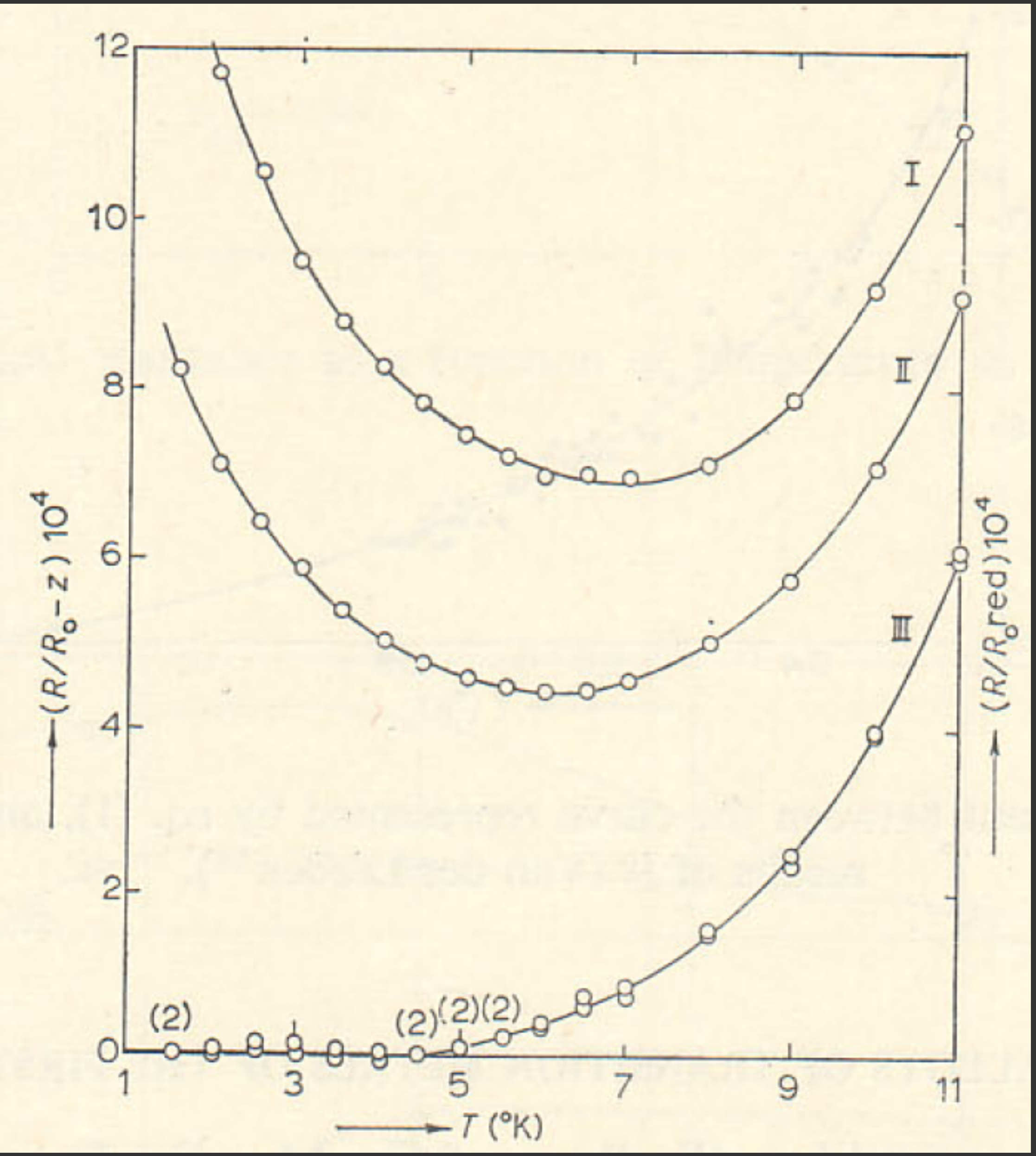


R-*T* curve of a normal metal at low *T*

$$\rho = \rho_0 + aT^2 + bT^5$$

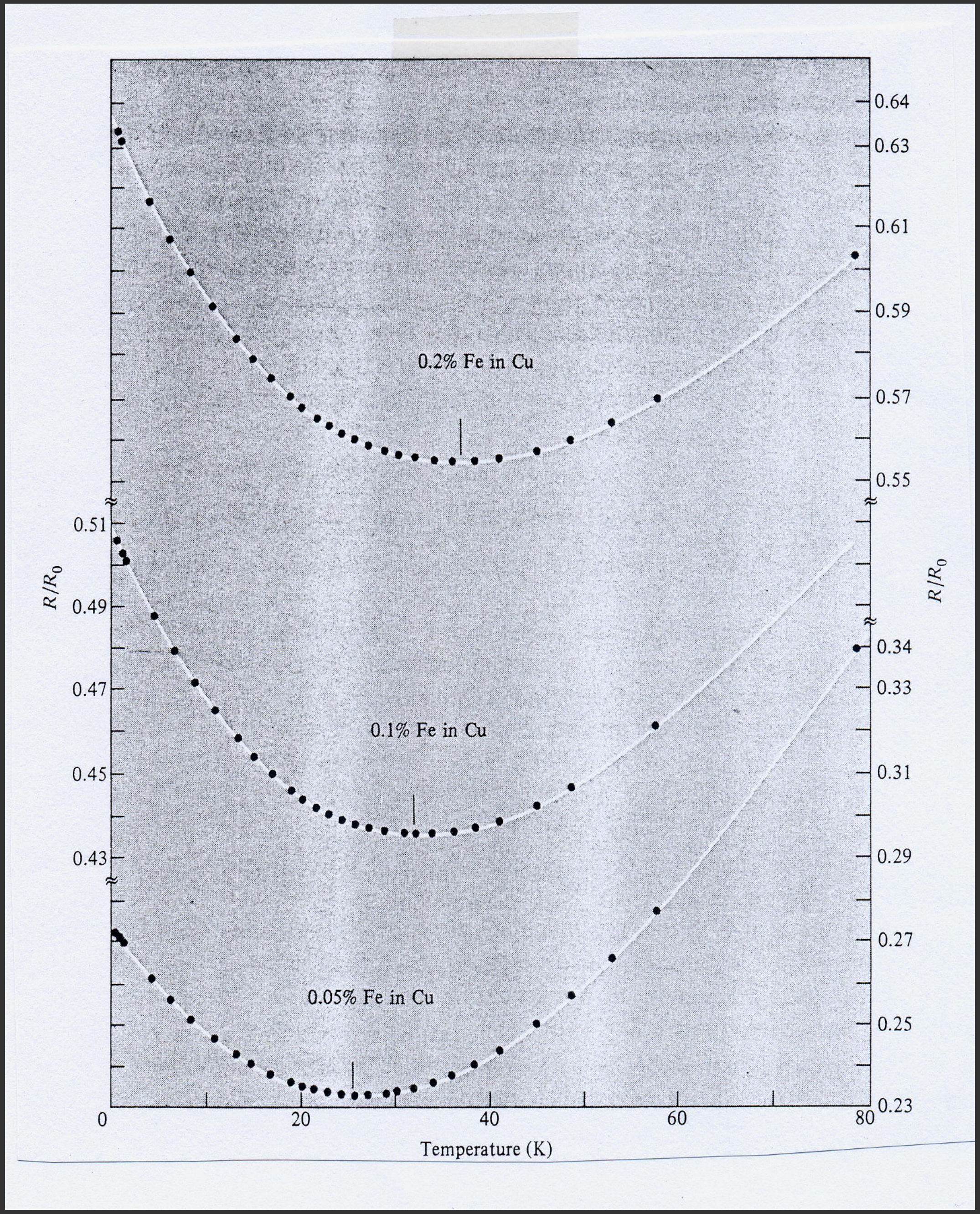
- ρ always decreases with reducing *T* due to smaller scattering rate.
- It is a monotonic function of temperature.

Resistivity minimum and impurity



- ρ-*T* curves of a series of Au wires with different level of purity (early 1960's)
- The resistivity minimum is induced by an increasing level of impurities!

The role of magnetic impurities



Controlled level of Fe impurities in Cu

Franck et al (1961)

- The higher the magnetic impurity density (c_{imp}), the higher the temperature that ρ reaches minimum. $T_{min} \sim c_{imp}{}^{1/5}$
- The higher the c_{imp}, the larger the depth of resistivity minimum. $\Delta\rho \sim c_{imp}$
- So it seems magnetic impurity causes the resistivity minimum phenomena

Magnetic impurities

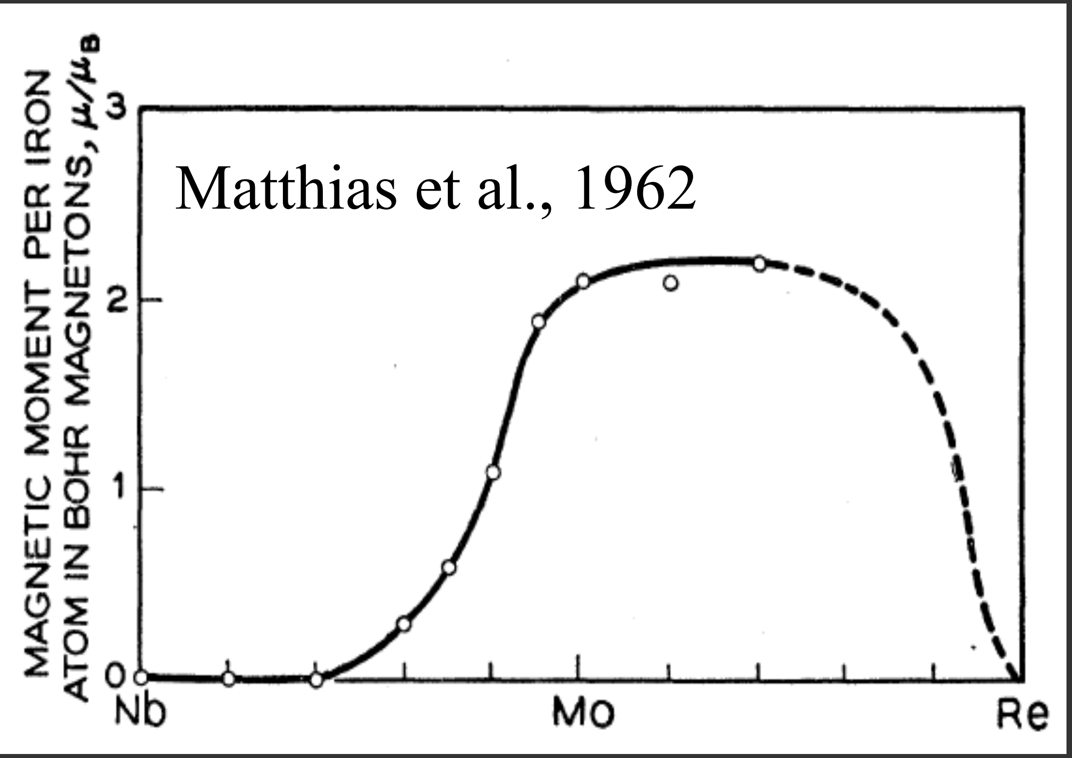
- The material systems that show the "resistivity minimum" phenomena are very dilute (1-10^3 ppm) "magnetic" elements (such as Ti, V, Cr, Mn, Fe, Co, Ni, …) dissolved into a host metals (such as Cu, Ag, Au, Al, Mg, Zn, ….).

Question: *what do you mean magnetic impurity?*

- Some of the impurities are not magnetic materials themselves.
- They are generally called "magnetic impurities" because they have a local magnetic moment when they are doped into *some* host metals.
- For some other combination of impurity and host metal, they may become non-magnetic, i.e., does not have a local moment.
- How to measure this experimentally?

Local moments of magnetic impurities

In early 1960's, Matthias's group at Bell Labs studied the magnetic behavior of an Fe atom dissolved in various Mo-Nb and Mo-Re alloys and their effect on superconductivity. Surprisingly, they found that:

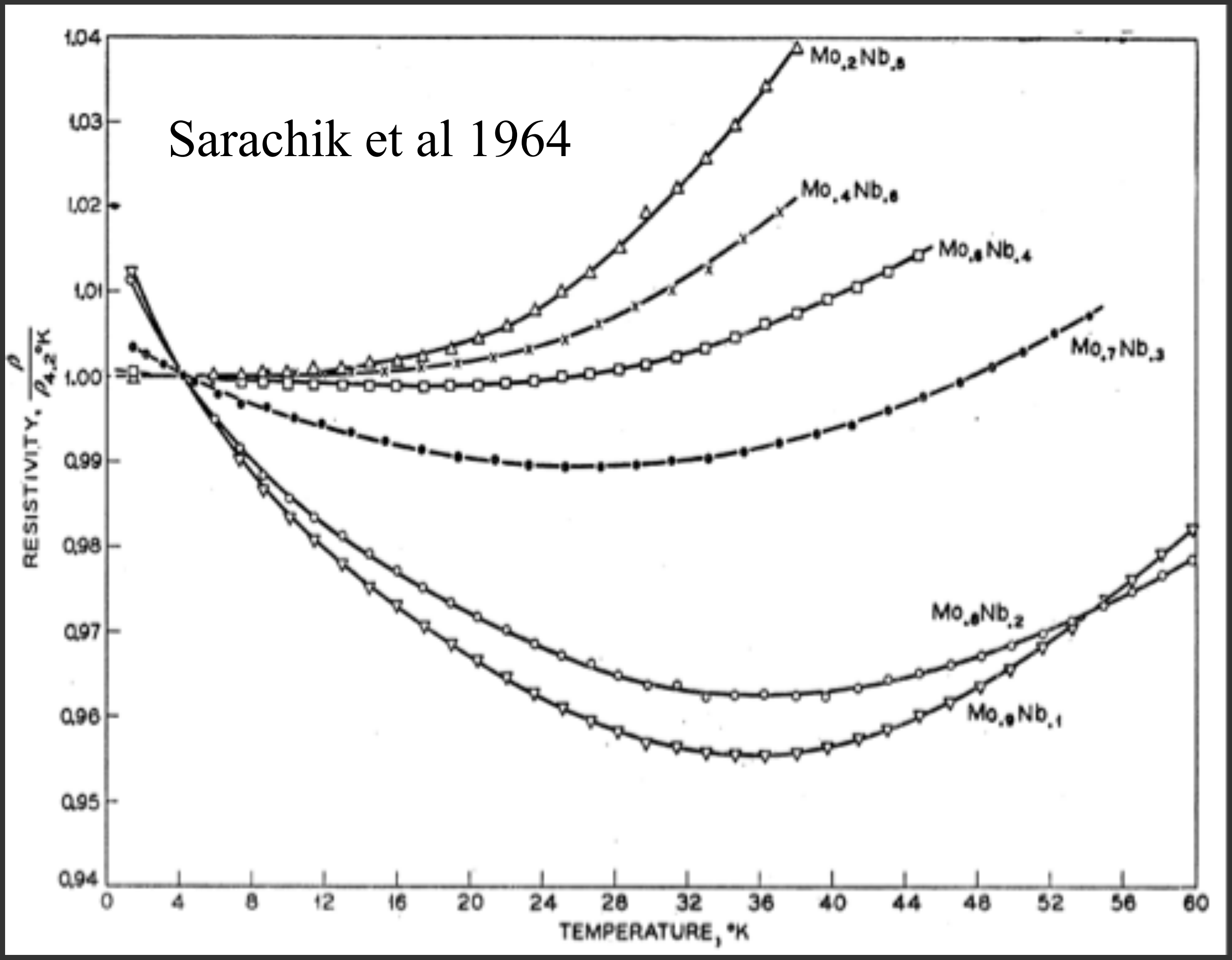


- In pure Nb, Fe dopants have no magnetic moment (Pauli susceptibility)
- Local magnetic moments of Fe starts to appear at about $Nb_{0.5}Mo_{0.5}$
- Strong local magnetic moment in pure Mo (Curie-Weiss susceptibility)
- Magnetic moment in all Mo-Re alloys

Conclusions:

- Even Fe dopants do not always have a local magnetic moment.
- It depends on what host metal they are dissolved in.

Magnetic moment and resistivity minimum



R-T curves of 1% Fe doped Mo-Nb alloys

Sarachik et al. (also at Bell labs) measured the resistivity of these alloys and found:

- no resistivity minimum in Nb-rich alloys.
- resistivity minimum starts to appear at about $Nb_{0.5}Mo_{0.5}$
- resistivity minimum becomes more pronounced when it is close to pure Mo

- There is a direction relationship between resistivity minimum phenomenon and the impurity magnetic moment.
- Resistivity minimum appears only when the impurities have magnetic moment!

Origin of impurity magnetic moment

- Why Fe dopants in Mo host metal have magnetic moments, but Fe dopants in Nb host metal have no magnetic moments?

- Where is the Fe magnetic moment from?

- From the unpaired *d* electrons.

- What above the *s* electrons?

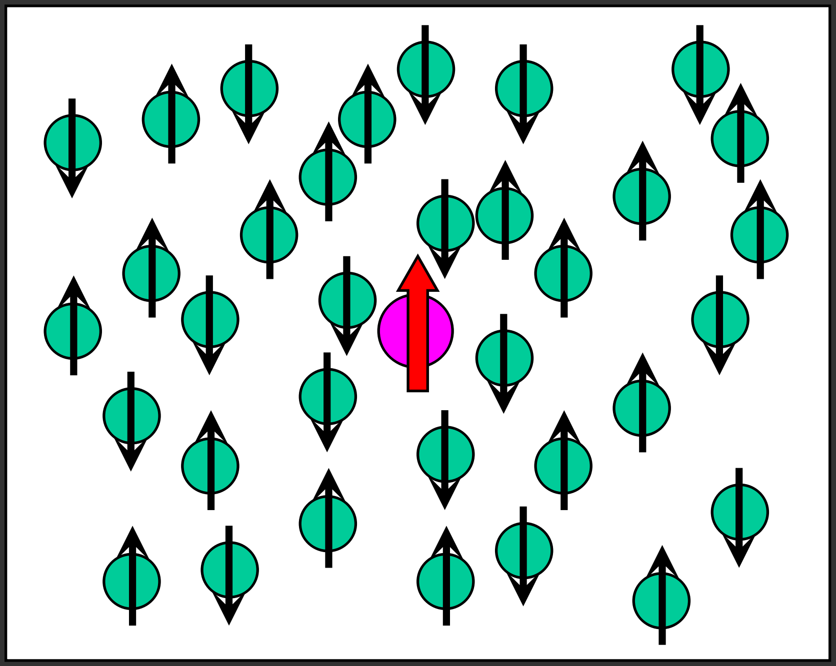
- They jump off the impurity and join the Fermi sea of the host metal.

- Will the *d* electrons jump off the impurity and join the Fermi sea as well?

- Will the electrons in the Fermi sea jump onto the impurity and form pair with the *d* electrons?

The Anderson Model (1961)

- To answer these questions, Anderson proposed a simple model and revealed the origin of the impurity local moment.
- He consider an impurity with *one* unpaired *d*-electron (spin ½) embedded in a Fermi sea.



The Anderson Model Hamiltonian:

$$H = \sum_{\sigma} \varepsilon_d n_{d,\sigma} + U n_{d,\uparrow} n_{d,\downarrow} + \sum_{k,\sigma} \varepsilon_k c^{\dagger}_{k,\sigma} c_{k,\sigma} + \sum_{k,\sigma} (V_k c^{\dagger}_{d,\sigma} c_{k,\sigma} + V^*_k c^{\dagger}_{k,\sigma} c_{d,\sigma})$$

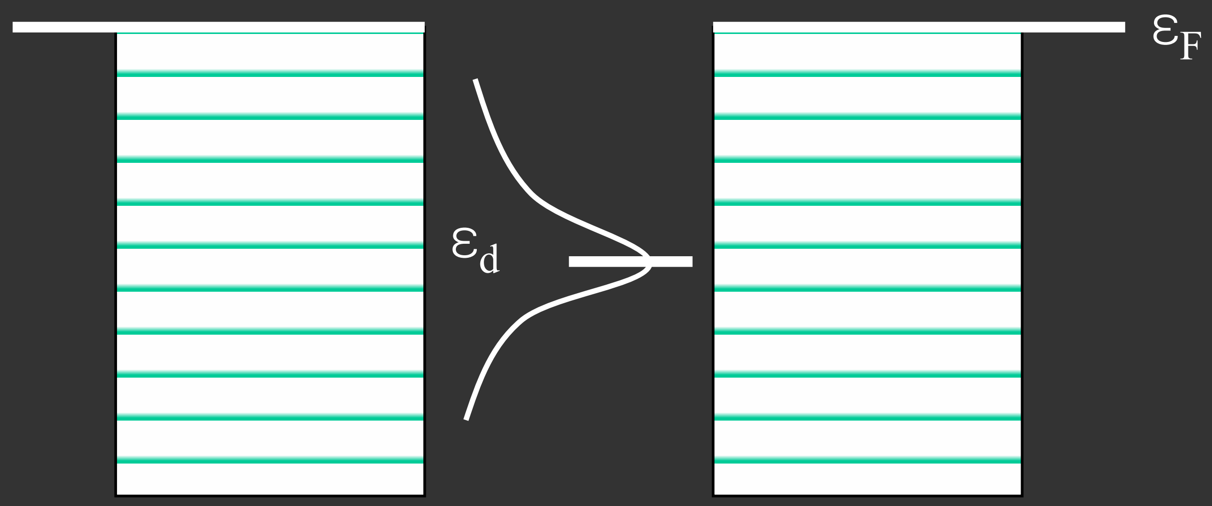
The Anderson Model (1961)

$$H = \sum_{\sigma} \varepsilon_d n_{d,\sigma} + U n_{d,\uparrow} n_{d,\downarrow} + \sum_{k,\sigma} \varepsilon_k c^\dagger_{k,\sigma} c_{k,\sigma} + \sum_{k,\sigma} (V_k c^\dagger_{d,\sigma} c_{k,\sigma} + V_k^* c^\dagger_{k,\sigma} c_{d,\sigma})$$

d electron level

conduction electrons level

Hybridization between *d* and conduction electron

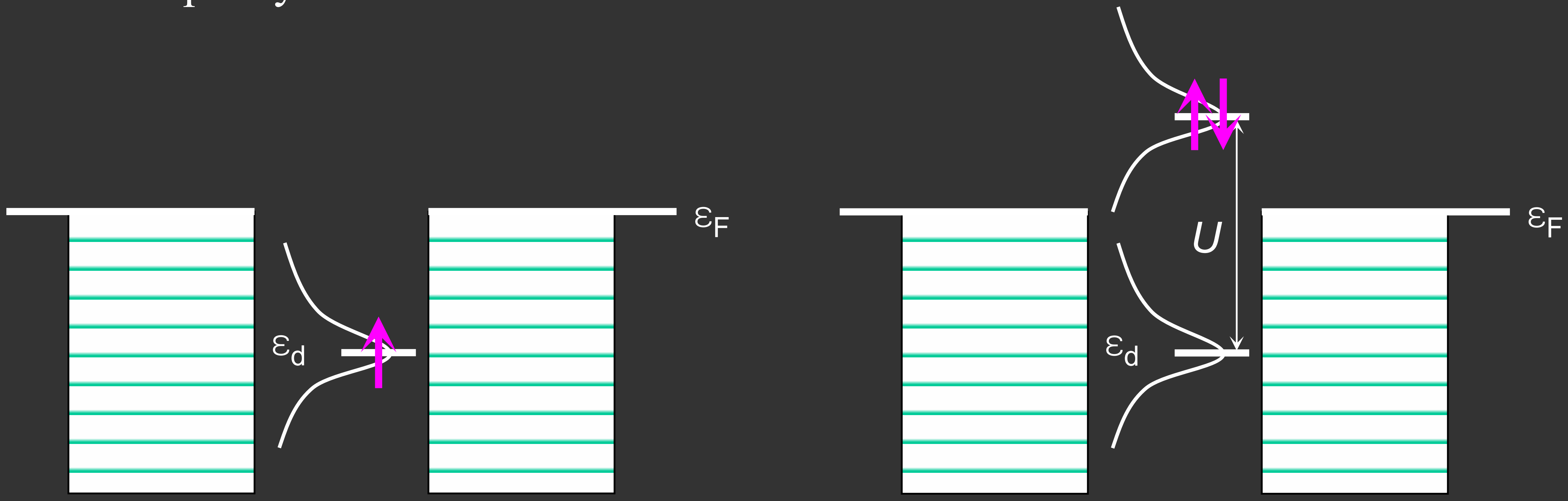


- The hybridization (V_k is the overlap or hybridization matrix element) between the conduction electrons and the impurity *d*-electron causes a shift and broadening of the local *d* level.

The interaction term

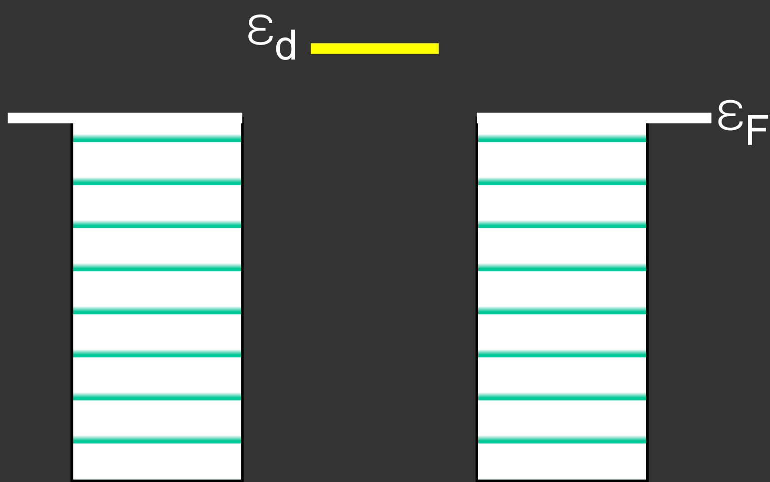
$$H = \sum_{\sigma} \varepsilon_d n_{d,\sigma} + U n_{d,\uparrow} n_{d,\downarrow} + \sum_{k,\sigma} \varepsilon_k c^{\dagger}_{k,\sigma} c_{k,\sigma} + \sum_{k,\sigma} (V_k c^{\dagger}_{d,\sigma} c_{k,\sigma} + V^*_k c^{\dagger}_{k,\sigma} c_{d,\sigma})$$

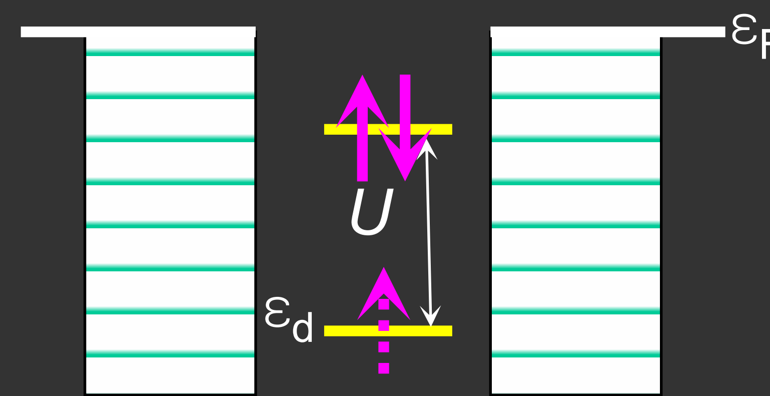
The most crucial feature of the Anderson model is the introduction of the interaction term, i.e., the Coulomb repulsion for two electrons occupying the same impurity site.



There are three different configurations for electrons on the impurity, depending on the relative level of ε_d and ε_F, and the strength of U.

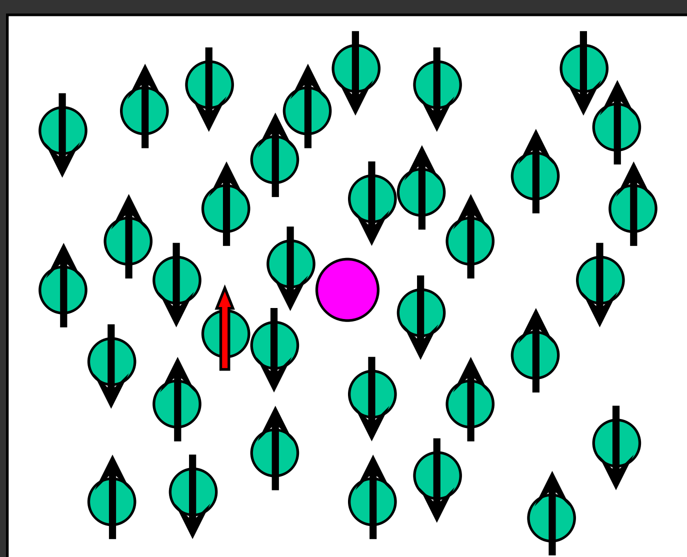
The non-magnetic situation

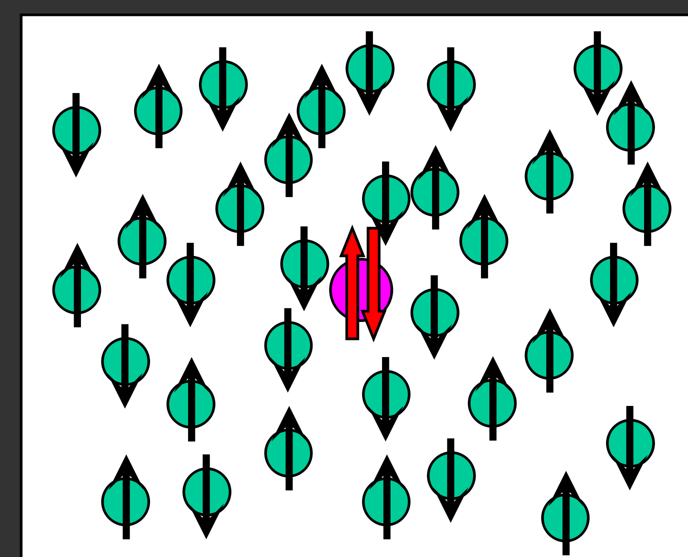




$\varepsilon_d > \varepsilon_F$

$\varepsilon_d < \varepsilon_F$ and $\varepsilon_d + U < \varepsilon_F$

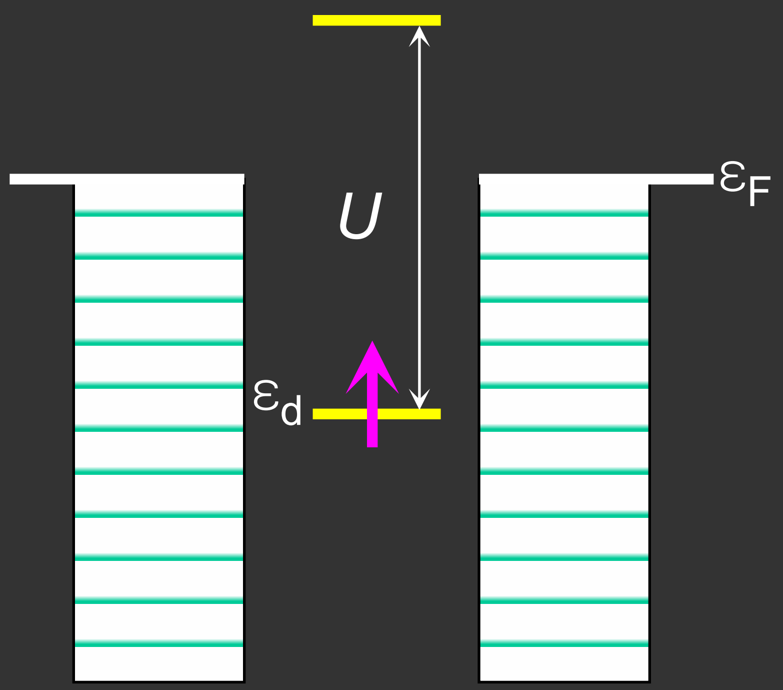




- The *d* electron jumps off the impurity and joins the Fermi sea
- $n_d = 0$, no local magnetic moment

- An electron from the Fermi sea jumps onto the impurity
- $n_d = 2$, no local magnetic moment

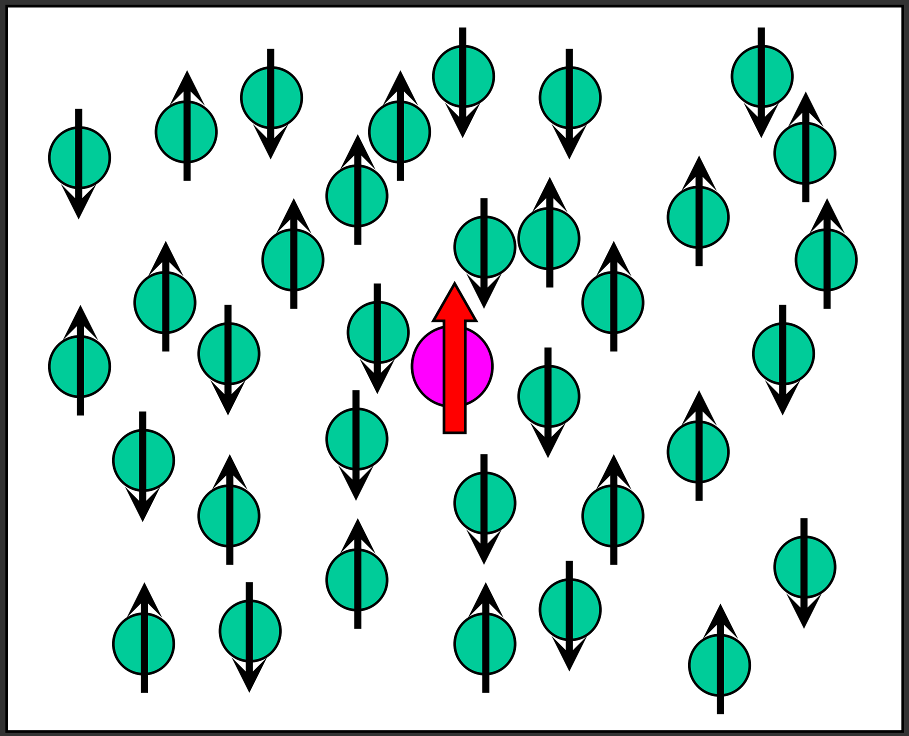
The magnetic situation



- $\varepsilon_d < \varepsilon_F$ but $\varepsilon_d + U > \varepsilon_F$
- The d electron cannot leave the impurity, and no more electron can join the impurity
- $n_d = 1$, local magnetic moment = spin ½
- Only in this situation can the impurity maintain its local moment.

- By considering the strong onsite Coulomb repulsion *U* between the *d*-electrons, the Anderson model gives a natural explanation for the presence or absence of localized moments on an impurities ion with partially filled *d* or *f* levels.
- This is a beautiful example of correlation effect in metals.

The resistivity minimum problem



- Now we are ready to tackle the resistivity minimum puzzle, which becomes a very well-defined question awaiting a solution.
- The system is dilute magnetic moments (spins) immersed in conduction electrons (a Fermi sea).
- It must be the local magnetic moments (spins) that affects electron conduction in metal, and causes a upturn in resistivity at low T.

The Kondo Model (1964)

Jun Kondo （近藤淳）

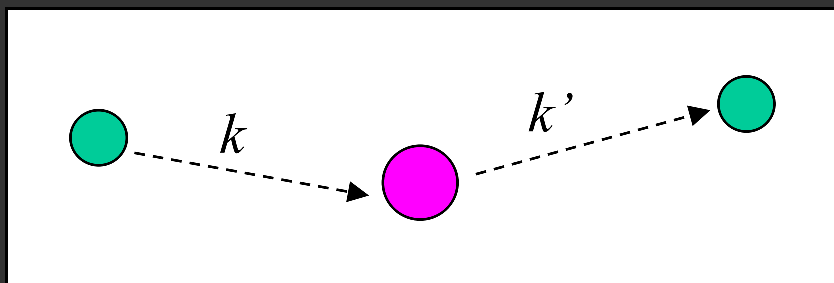
Kondo's line of thought:

- Resistivity is caused by the scattering of conduction electrons.
- At low T the scattering is mainly impurity scattering
- In the resistivity minimum case it is the scattering by magnetic impurities (spins), so scattering probability is related to the interaction between the conduction electrons and the impurity d electron *spins*, not charges.

- The resistivity minimum depth is roughly proportional to c_{imp}, so the scattering is a single impurity effect, and we simply add their contributions together.
- No need to consider interaction between impurity spins (in the dilute limit).

The problem is reduced to *spin-dependent scattering rate due to a single impurity!*

Impurity scattering *without* spin interaction



Scattering from state k to k' by potential $V(r)$,
scattering matrix element: $V_{kk'} = \langle k \mid V(r) \mid k' \rangle$

$$H = \sum_{k,\sigma} \varepsilon_k c^\dagger_{k,\sigma} c_{k,\sigma} + \sum_{k,k',\sigma} V_{k,k'} c^\dagger_{k',\sigma} c_{k,\sigma}$$

Treat $V_{kk'}$ as perturbation,
first order scattering:

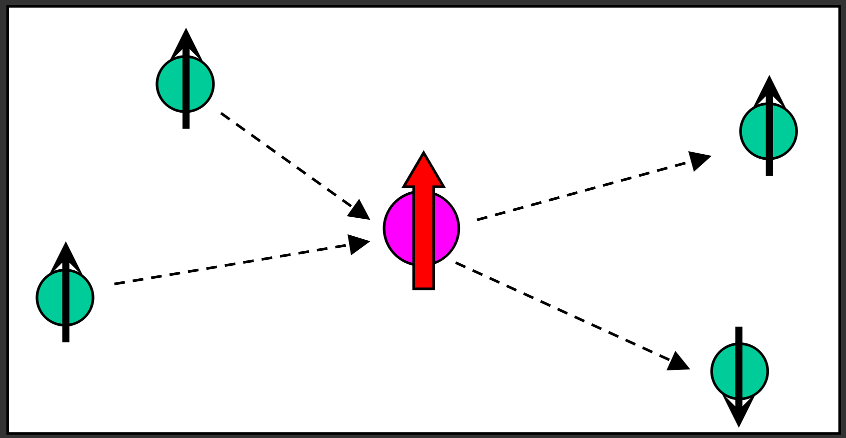
$$\frac{1}{\tau_k} = 2\pi c_{imp} \int \delta(\varepsilon_k - \varepsilon_{k'}) |V_{kk'}|^2 (1 - cos\theta') \frac{dk'}{(2\pi)^3}$$

impurity density; elastic scattering; scattering potential; back scattering dominant

Assume V is symmetric and k-independent: $R \sim c_{imp} V^2$

It's proportional to c_{imp} and V but independent of T, called *potential scattering*.

The Kondo model: impurity scattering *with* spin interaction

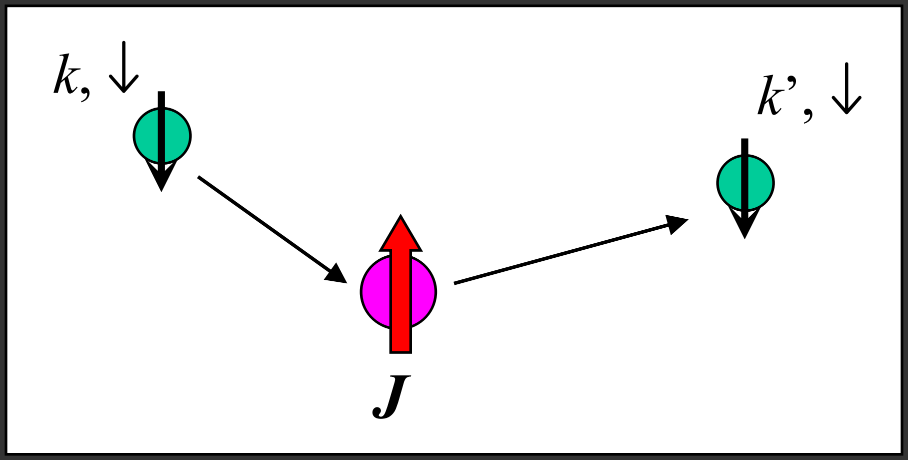


The interaction between the spin of the impurity and the conduction electrons is exchange interaction, which can be characterized by an exchange integral *J*.

Kondo Model: $$H = \sum_{k,\sigma} \varepsilon_k c^\dagger_{k,\sigma} c_{k,\sigma} + J \sum_{k,k',\sigma,\sigma'} (c^\dagger_{k',\sigma'} \vec{\sigma}_{\sigma,\sigma'} c_{k,\sigma}) \cdot \vec{S}$$

- Scattering from state (k, σ) to state (k', σ') by exchange interaction.
- σ and S are conduction electron and impurity spins respectively.
- *J* can be treated as a perturbation using Born Approximation.

First order direct scattering



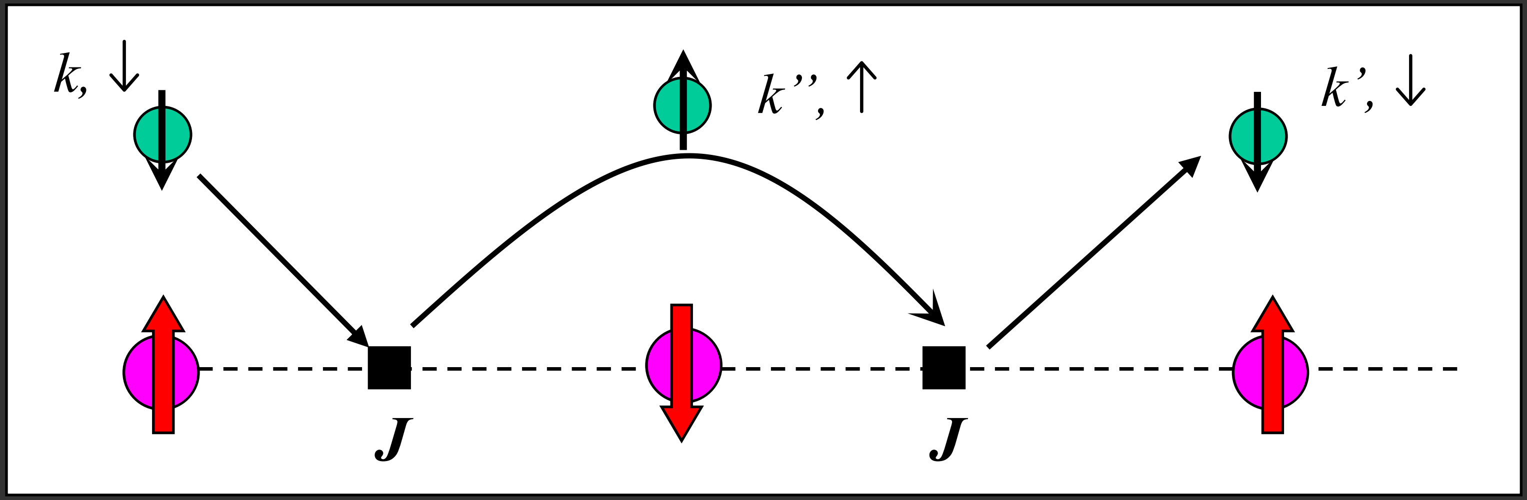
$k, \downarrow$ ⟹ $k', \downarrow$

- In first order scattering, the electron with momentum k and spin $\downarrow$ is directly scattered to a state with k' and $\downarrow$.
- The impurity spin only provides a scattering potential, its spin is not involved in the scattering process.
- The scattering is very similar to the potential scattering, except that now the scattering matrix element $W_1 \sim J$
- The total scattering rate is thus $1/\tau \sim c_{imp}J^2$.
- It's independent of T, cannot explain the resistivity minimum.

Second order indirect scattering

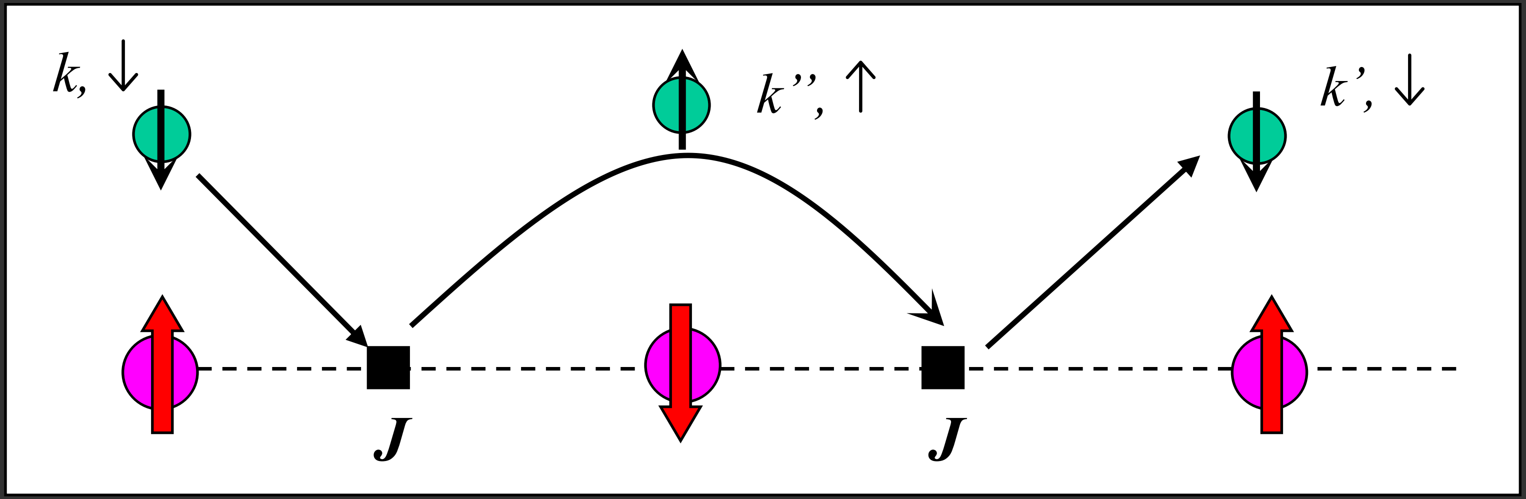
Kondo extended the perturbation calculation to the second order scattering, i.e., scattering through intermediate states that the impurity spin is involved.

- We still consider the scattering from initial state (*k*, ↓) to final state (*k'*, ↓).



- This diagram is only one of the 2nd order scatterings that involves the flip of impurity spin.
- First a conduction electron (*k*, ↓) is scattered into a state (*k''*, ↑), meanwhile flip the impurity spin is flipped from ↑ to ↓.
- This is only an intermediate state, there is a further scattering process to arrive at the final state (*k'*,↓), in which the impurity spin is reversed back to ↑.

Second order indirect scattering



The scattering matrix element for this whole process is:

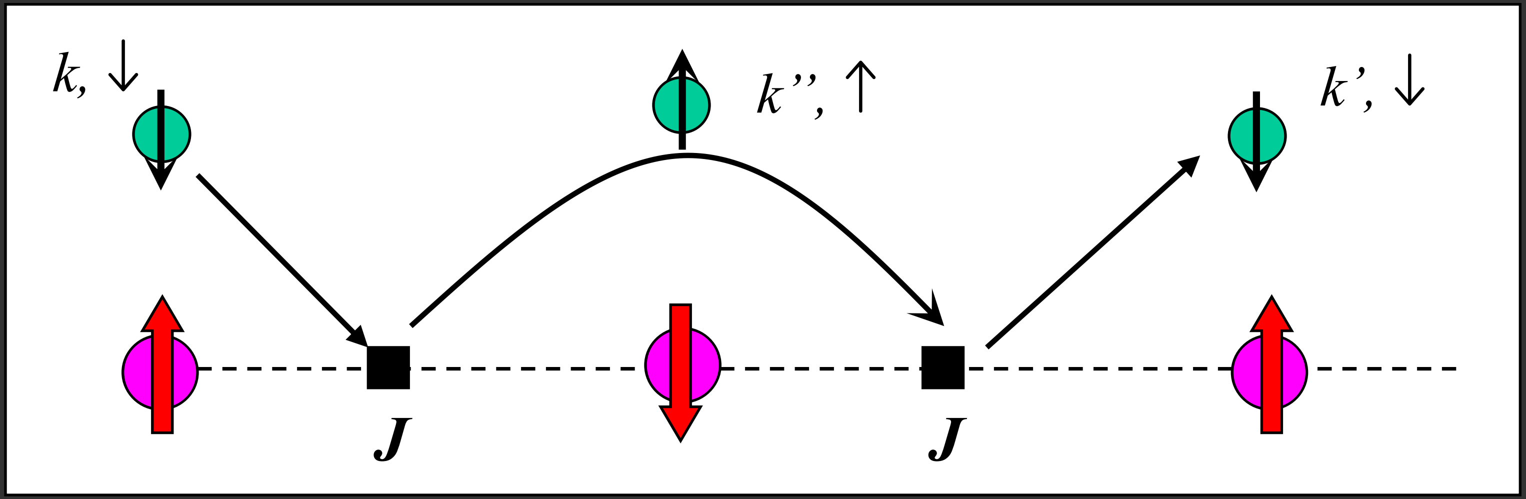
$$\sum_{k''} J(k\downarrow, d\uparrow \rightarrow k''\uparrow, d\downarrow)\cdot J(k''\uparrow, d\downarrow \rightarrow k'\downarrow, d\uparrow)\frac{1-f_{k''}}{\varepsilon_k-\varepsilon_{k''}}$$

Assume that *J* is a constant, replace the sum of *k*'' by an integral over $\varepsilon_{k''}$:

$$J^2 D\int \frac{1-f_{k''}}{\varepsilon_k-\varepsilon_{k''}}d\varepsilon_{k''} = J^2 D\int_{\varepsilon_F}^{\varepsilon_D} \frac{1}{\varepsilon_k-\varepsilon_{k''}}d\varepsilon_{k''}$$

D is the density of state assumed to be constant. The scattered electrons *k*'' can only take states between ε_F and ε_D, ε_D is the top of the band.

Second order indirect scattering



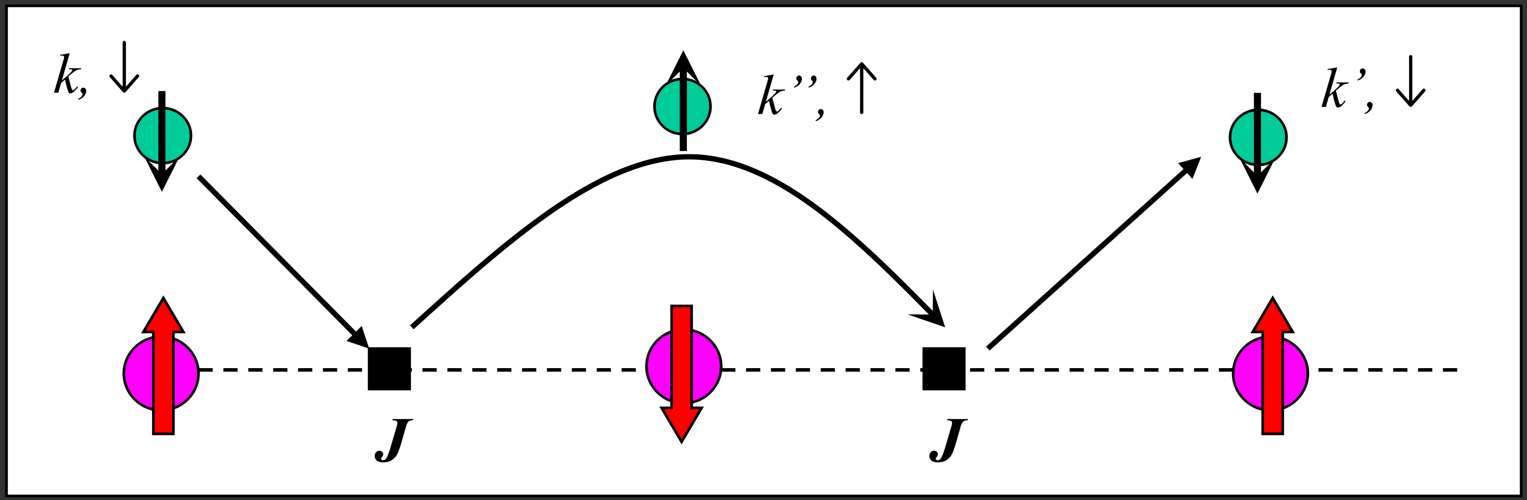
Then the scattering matrix element is: $$W_2 = J^2 D \cdot \log(|\frac{\varepsilon_k - \varepsilon_F}{\varepsilon_k - \varepsilon_D}|)$$

- The scattered electrons only lie within a window of $k_B T$ about ε_F so $|\varepsilon_k - \varepsilon_F| \sim k_B T$.
- For half filled band, $|\varepsilon_k - \varepsilon_D| \sim$ ½ bandwidth $\sim \varepsilon_F$

Thus we have: $$W_2 = J^2 D \cdot \log\frac{k_B T}{\varepsilon_F}$$

The total scattering matrix element: $$W_{total} = W_1 + W_2 = J + J^2 D \cdot \log\frac{k_B T}{\varepsilon_F}$$

Second order indirect scattering



Total scattering rate from one impurity: $$\frac{1}{\tau} = |W_1 + W_2|^2 \sim J^2 + 2J^3 D \cdot \log\frac{k_B T}{\varepsilon_F}$$

Total resistivity: $$R(T) = R_0(1 + 2JD \cdot \log\frac{k_B T}{\varepsilon_F})$$ $R_0 \sim c_{imp} J^2$

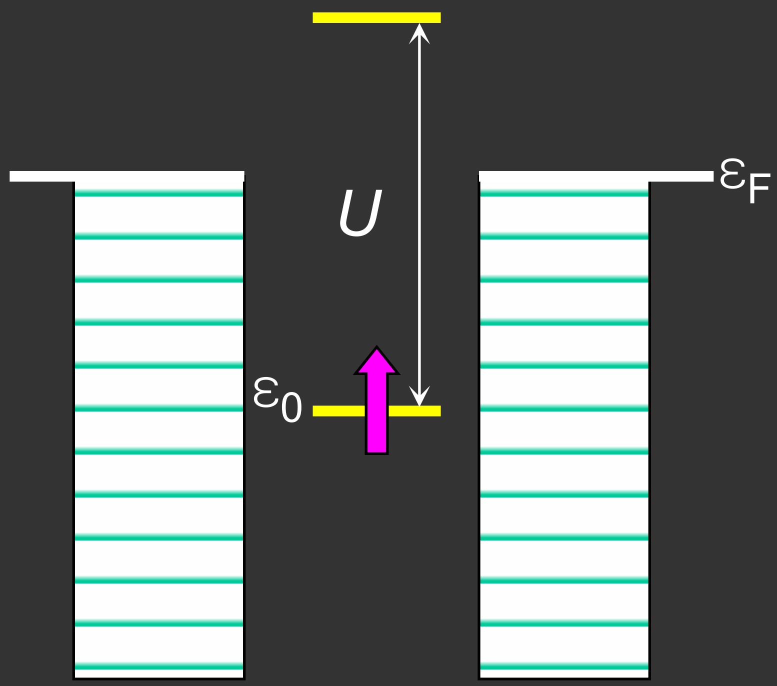
The T-dependence comes from the second order correction $J \log T$ term.

- $J > 0$, FM exchange, $J \log T$ decreases with decreasing T, no resistivity minimum
- $J < 0$, antiferromagnetic exchange, $J \log T$ increases with decreasing T, this is opposite to the other scattering trend, so there is a *resistivity minimum!*

Why is *J* negative?

- Kondo's theory told us that in order to have the resistivity minimum, the exchange interaction between impurity local moment and conduction electrons should be antiferromagnetic, i.e., $J < 0$.

- Why is $J < 0$? What is the origin of the exchange?

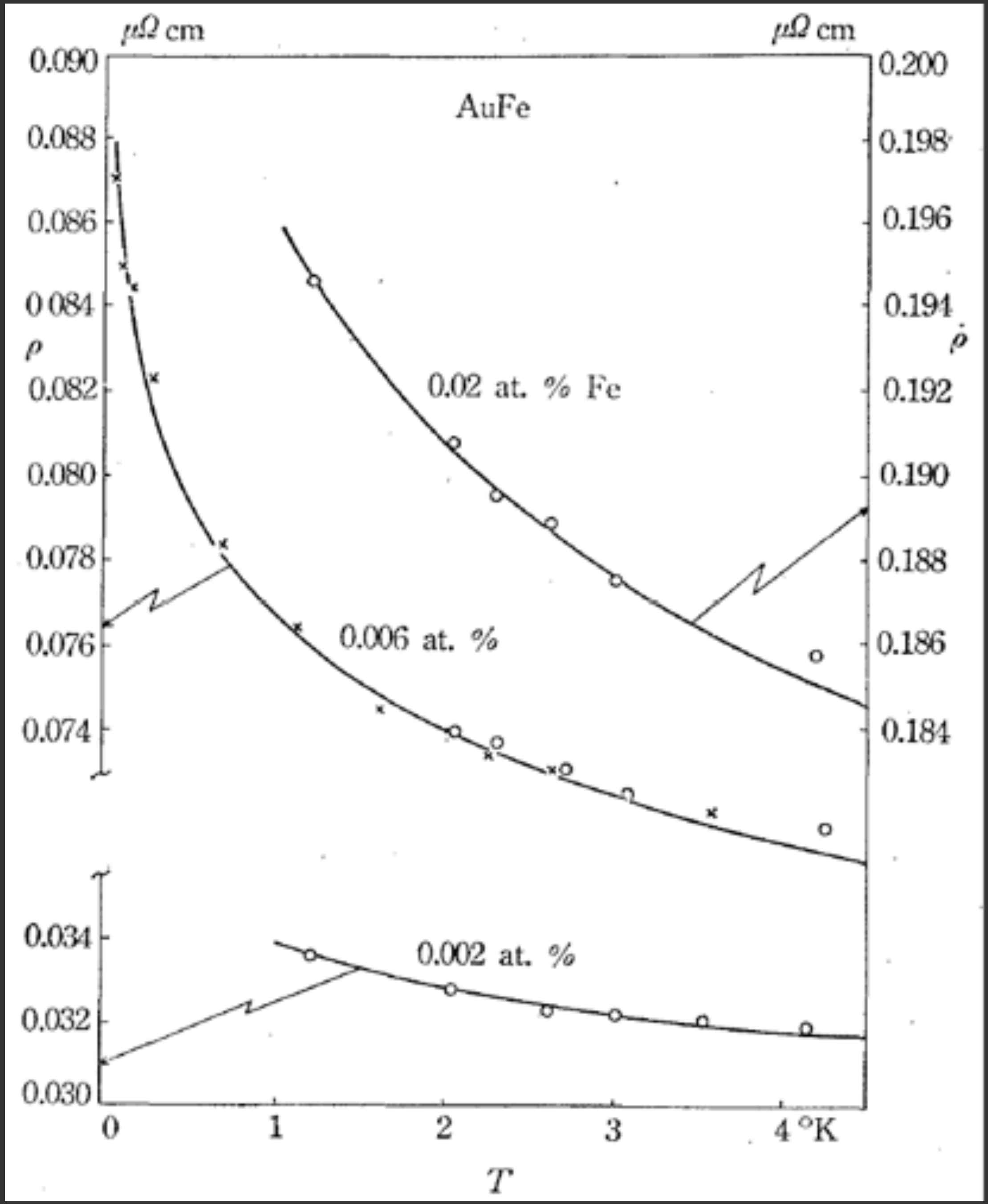


- Roughly speaking it is very similar to the superexchange mechnism.
- Hybridization of *s*-*d* electrons allows hopping via an intermediate state with the impurity site doubly occupied.

- The strength of AF exchange:

$$J \sim |v|^2 \left(\frac{1}{E_F - \varepsilon_0} + \frac{1}{\varepsilon_0 + U - E_F} \right)$$

Experimental test of Kondo's formula



ρ-T of Au-Fe alloys (symbols) and Kondo's theoretical fit (curves)

Comparison with ρ-T curves:

- Kondo formula: $\rho(T) = a - b \log T$
- Very good agreement with experiments

Comparison with T_{min} vs. c_{imp}

- Total resistivity formula:

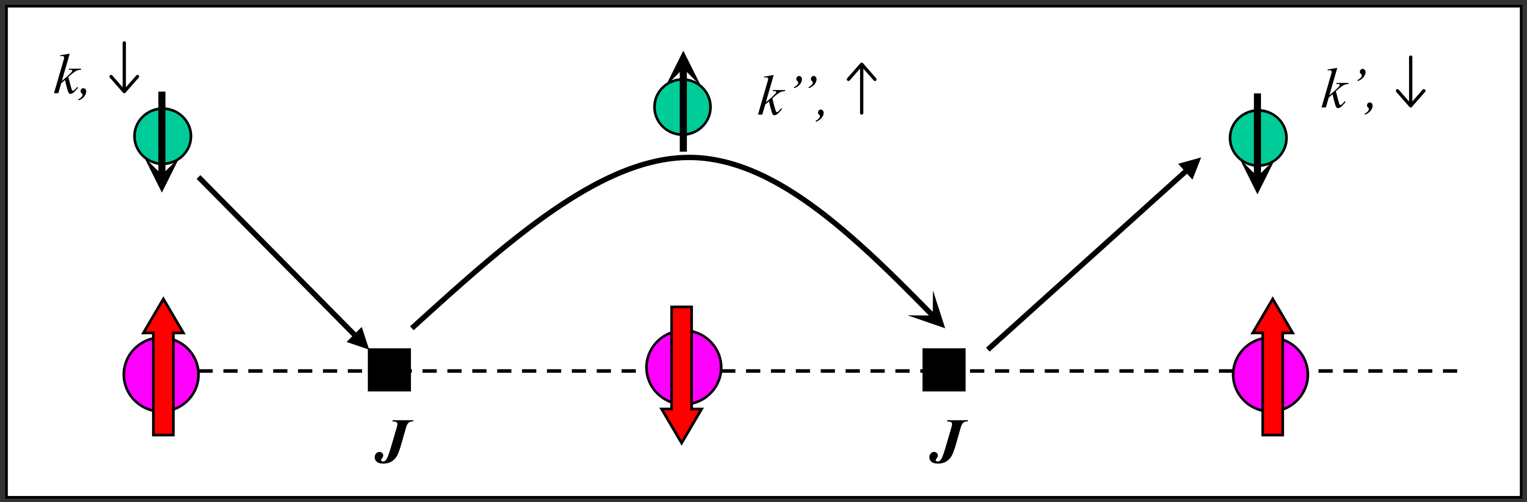
  $\rho(T) = \rho_0 + \alpha T^2 + \beta T^5 - \gamma c_{imp} \log T$

- The T^2 term is usually small, then:

  $T_{min} \sim c_{imp}^{1/5}$ $(d\rho/dT = 0)$

- Excellent agreement with experiments

The Kondo effect



- The comparison unambiguously confirmed the correctness of Kondo's theory
- The resistivity minimum puzzle in dilute magnetic alloys is finally solved.
- It is due to the second order scattering (that involves the flip of impurity spin) of conduction electrons by antiferromagnetic exchange interaction with the impurity spin.
- Because of Kondo's crucial contribution in solving the resistivity minimum puzzle, *this effect is generally known as the "Kondo Effect".*

The Kondo Problem

Kondo solved a big puzzle, but created an even bigger puzzle …

Kondo resistivity formula: $\rho(T) = a - b \log T$

When $T \to 0$, $\rho \to \infty$, the resistivity diverges as T approaches zero.

- It means when T approaches zero, the 2nd order perturbation (the logarithmic term) is much larger than the 1st order perturbation, so perturbation theory cannot apply in this regime.
- Kondo's perturbation solution is correct only at relatively high T.
- We need to find a new theory, most likely a non-perturbative theory to treat the Kondo effect at low T.
- This was known as the Kondo Problem.

The Kondo Problem

- The Kondo problem is a very well-defined (single impurity spin interacting with conduction electrons) and deceptively simple problem (find an increasing but non-divergent resistivity at low T), yet it is extremely difficult to solve.

- People's initial trial was to go to higher order perturbation and see if the log T divergence will go away (Abrikosov 1965, Kondo 1969).
- By summing up higher order terms, they found that Kondo's log T solution is valid above a characteristic temperature, called the Kondo temperature T_K.

$$T_k \sim W e^{-1/JD}$$

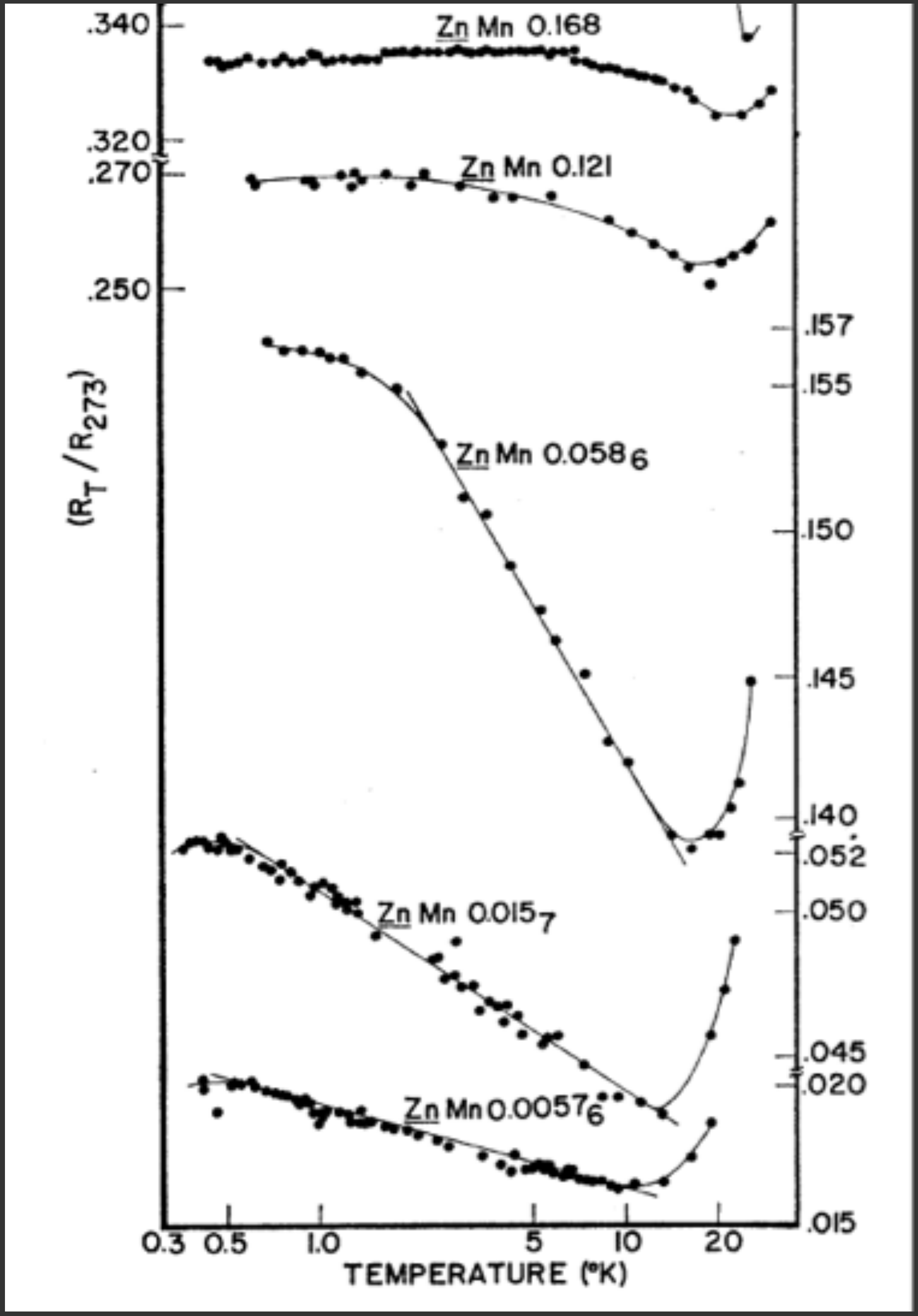
W is the bandwidth of the conduction band, J is antiferromagnetic exchange ($J > 0$ in this convention), D is the conduction electron DOS

The Kondo Temperature

$$T_k \sim W e^{-1/JD}$$

- Below T_K, the perturbation method totally breaks down.
- The reason is at such low temperature, the scattering between the conduction electrons and impurity spin is purely quantum mechanical.
- The microscopic states of the conduction electrons become correlated through their spin-flip scatterings with the impurity.
- This is intrinsically a strongly correlated quantum many body problem.
- Therefore we can not describe them starting from single particle Fermi liquid state and perturb it.
- The Kondo problem attracted a lot of attention because it can be used as a test ground for the many-body theory under development at that time.

Resistivity below T_K



Hedgcock et al., 1967

- Mn doped Zn resistivity down to very low temperature
- Resistivity is logarithmic above T_K, but deviates from the log behavior at $T < T_K$
- At very low T the resistivity saturates at a certain value, or even starts to decreases with a T^2 behavior.
- What happens at $T << T_K$?

Anderson's Poor man's scaling (1969)

- In 1969, Anderson used the scaling technique to address the Kondo problem.

- What is the origin of the logarithmic diverging?

The 2nd order scattering matrix element is: $$W_2 = J^2 D \cdot \log(|\frac{\varepsilon_k - \varepsilon_F}{\varepsilon_k - \varepsilon_D}|)$$

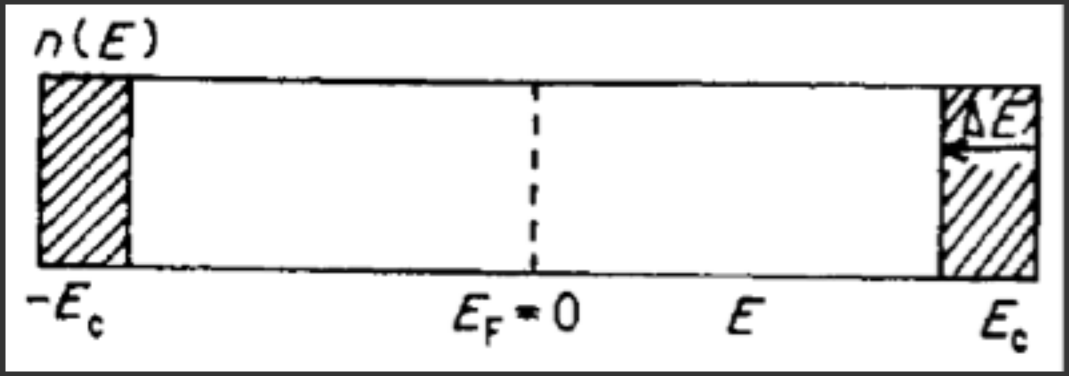
- The dominant contributions come from electrons lying within $k_B T$ about ε_F.

Thus we have: $$W_2 = J^2 D \cdot \log \frac{k_B T}{\varepsilon_F}$$

- The logarithmic divergence is a consequence of the sharpness of Fermi surface at low temperatures.

Anderson's Poor man's scaling (1969)

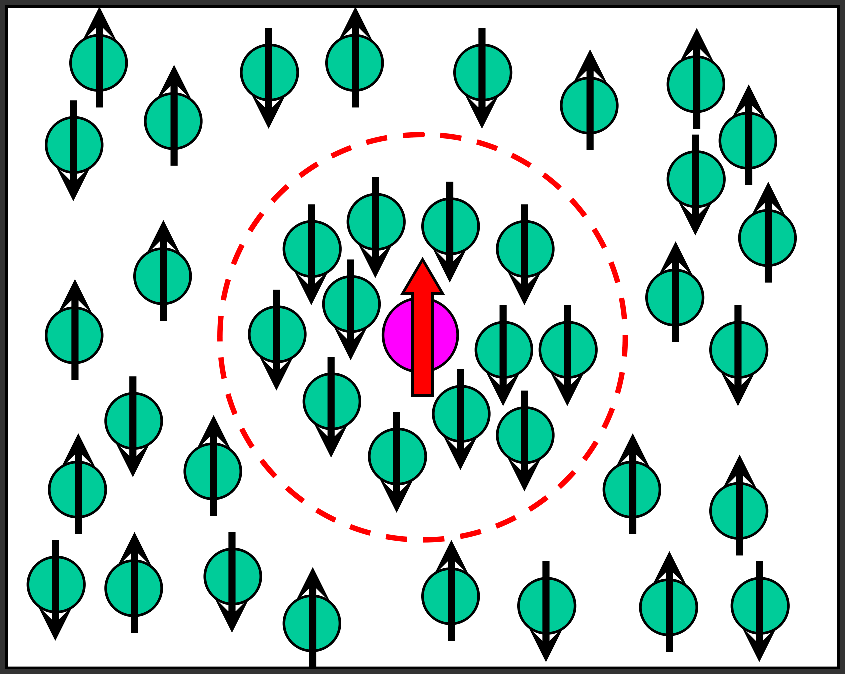
- The basic idea of scaling is to find a new, solvable low energy effective model by eliminating the high energy excitations via the change of energy scales.



- In treating the Kondo problem, Anderson successively eliminated the high energy states of the conduction electrons using perturbation techniques (i.e., continuously reduce the conduction electron bandwidth).

- The scaling analysis shows that for antiferromagnetic exchange interaction, the scattering between the conduction electrons and the impurity spin becomes stronger as the conduction electron energy approaches the Fermi level ($\varepsilon_k \rightarrow \varepsilon_F$), or when the temperature approaches zero ($T \rightarrow 0$).

The ground state of the Kondo problem



- By analyzing the asymptotic behavior of the scaling laws, Anderson made the conjecture that for $T << T_K$ ($T \to 0$), $J \to \infty$, the impurity spin and the conduction electron spins form a bound state with $S = 0$.
- So the impurity spin is screened by conduction electrons via spin-flip scattering, and won't be seen directly by other conduction electrons (similar to the screening of charged Coulomb potential).
- Therefore the scattering process becomes spin-independent, could be described by Landau FL theory.

Wilson’s Numerical renormalization method

- Anderson’s scaling conjecture solved the Kondo divergence problem, but it is still a conjecture because the perturbation method breaks down as $T \to T_K$.
- The final solution of the Kondo problem was provided by Ken Wilson.

Kenneth G. Wilson

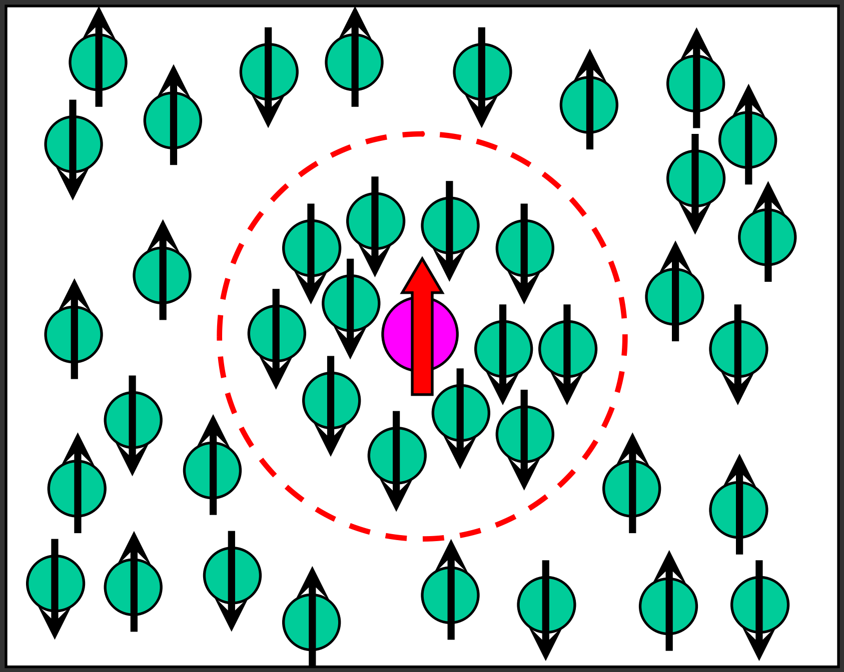
- PhD from Caltech (1961), advisor Murray Gell-Mann.
- Harvard Junior Fellow from 1961-1963.
- Published only one paper during PhD and postdoc.
- Joined Cornell in 1963 as a junior faculty
- Published only three pretty bad papers between 1963 and 1969, but still made tenure.
- Published his seminal work in 1970, one of the most important breakthroughs in theoretical physics.
- 1982 Nobel prize “for his theory for critical phenomena in connection with phase transitions”
- Quasi "by the way" solved the Kondo problem.

Wilson's renormalization treatment of the Kondo Problem

- He proved Anderson's conjecture that for the $S = ½$ case, the antiferromagnetic coupling $J \rightarrow \infty$ as $W \rightarrow 0$.
- So the ground state of the Kondo system is a bound state, the spin of the impurity is screened by the conduction electrons.
- Wilson got the effective Hamiltonian and calculated the low temperature thermodynamic behavior, which agrees well with experiments.
- Using Wilson's NRG method, the behavior of resistivity can be calculated for a wide range of temperatures, and quantitatively agrees with experiments.

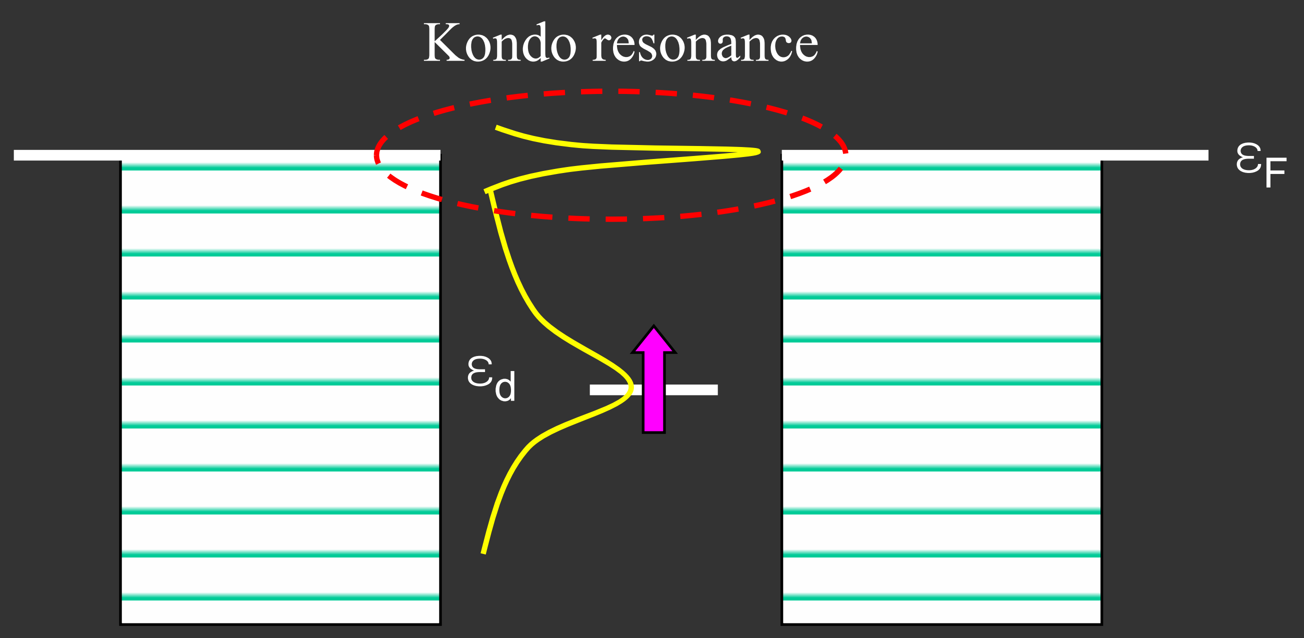
Wilson's numerical renormalization method gave an exact solution of the Kondo problem, the puzzle is totally solved!

Kondo Screening Cloud



- For $T < T_k$, the conduction electrons form a correlated many-body ground state to screen the impurity spin and yield a total spin 0.
- This has been confirmed by susceptibility measurement. HOW?
- The spatial scale of the Kondo screening cloud is $\xi = \hbar v_F / k_B T_k$.
- In a normal metal $\xi \sim 1\ \mu$m, there is a huge number of electrons in it.
- So it is a true many-body collective mode.

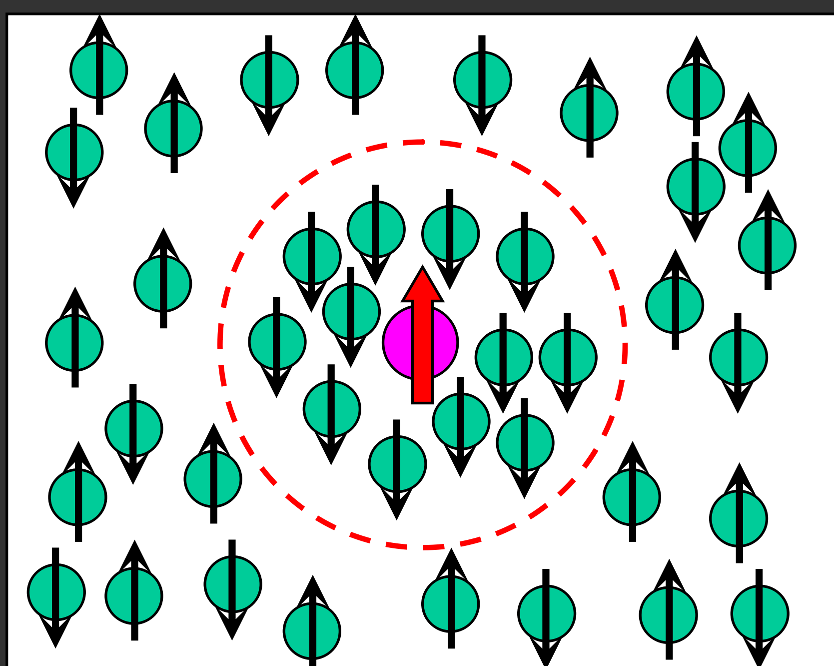
Kondo Resonance

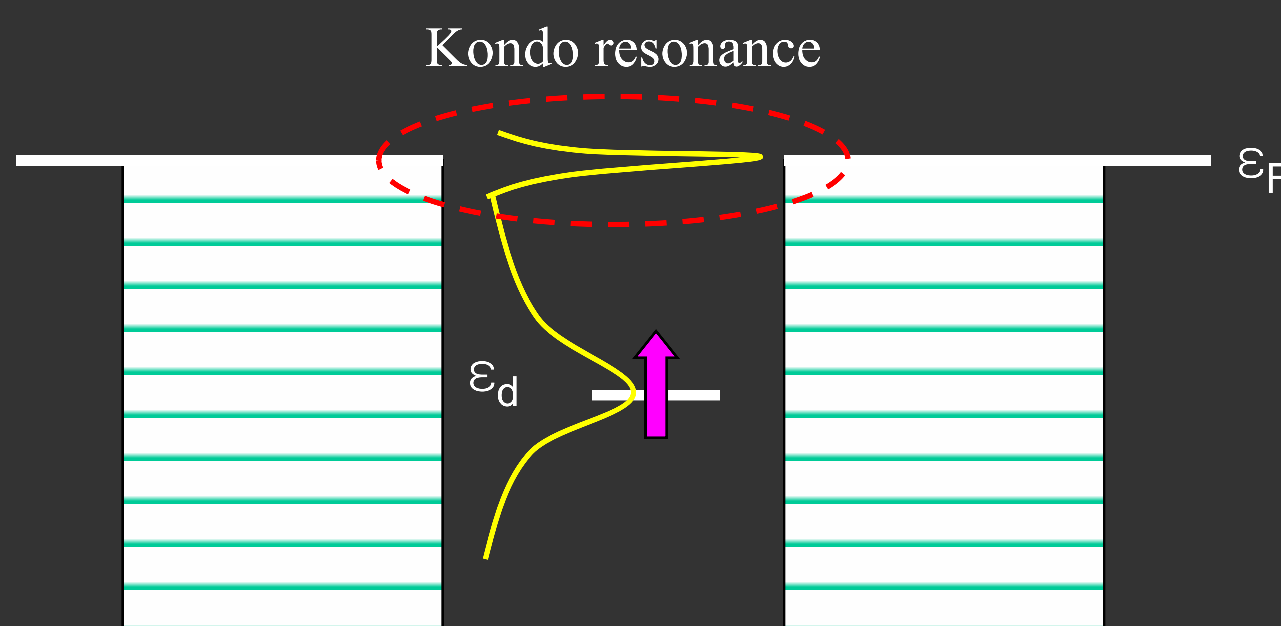


- Below T_K, a strongly enhanced local DOS pinned at ε_F appears on the impurity.
- Fermi level conduction electrons form a dense screening cloudy around the impurity, collectively they give exactly one spin to compensate the impurity spin.
- This sharp resonance, called the Kondo resonance, is one of the most dramatic consequences of the Kondo interaction.
- The Kondo resonance has a Lorentzian line-shape with width $\sim k_B T_k$, and disappears for $T > T_k$.

STM studies of single atom Kondo effect

- The simplest Kondo system is a single magnetic impurity atom in a non-magnetic metal host.
- The most important spectroscopic feature of the many-body Kondo effect is the Kondo resonance at ε_F on the impurity site at $T < T_k$.
- STM is the perfect probe for measuring the LDOS of a single magnetic impurity.

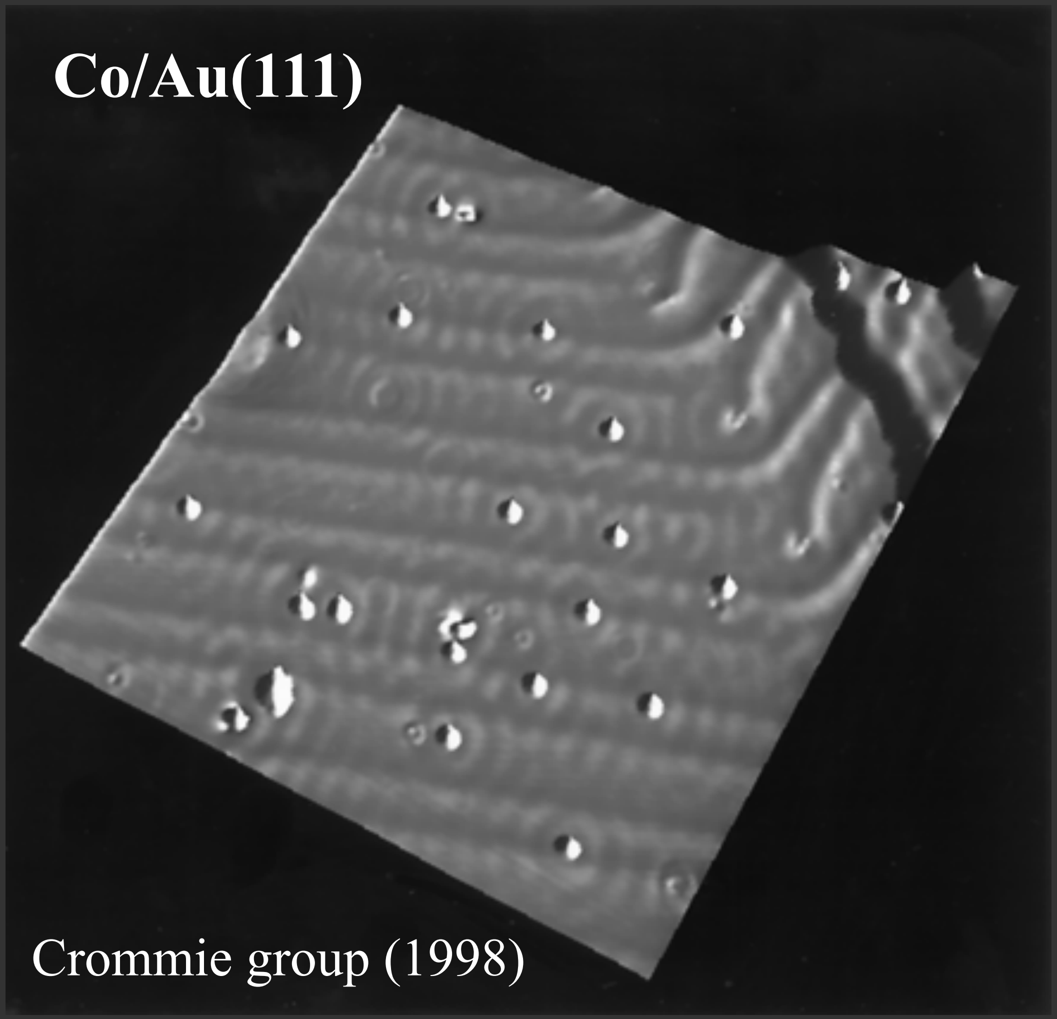


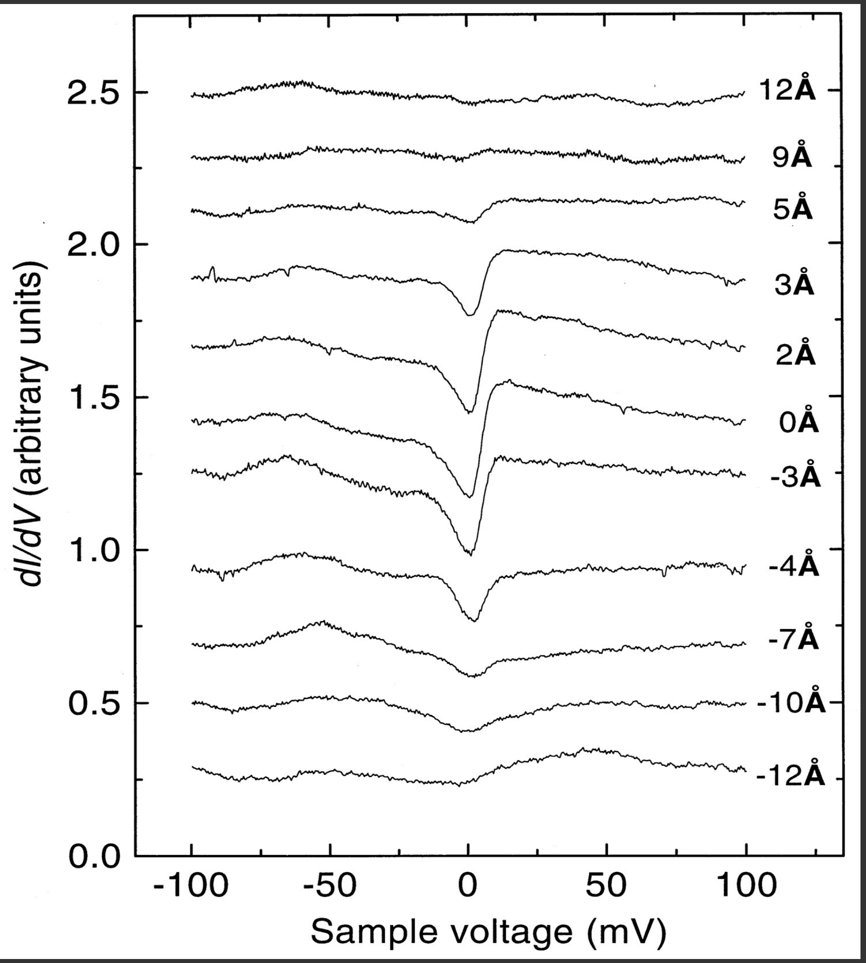


STM can see the impurity atom

STS can detect the Kondo resonance

Kondo effect of a single magnetic atom

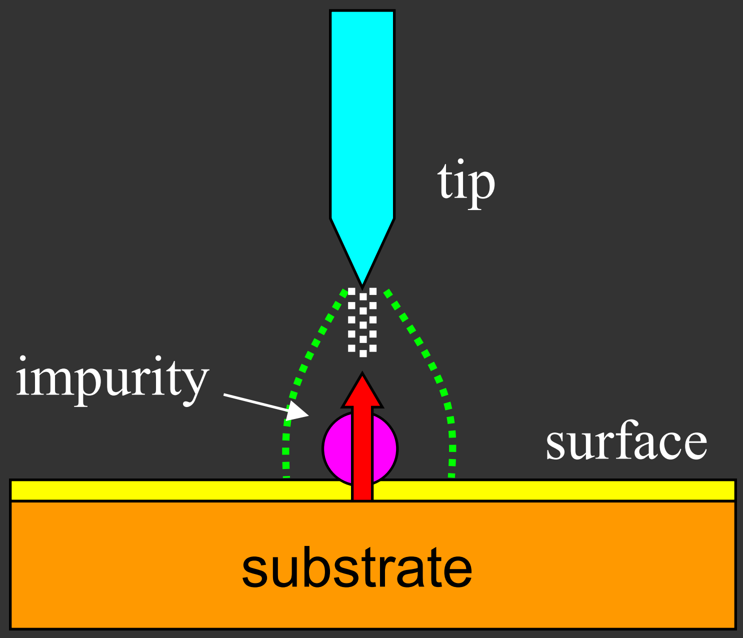


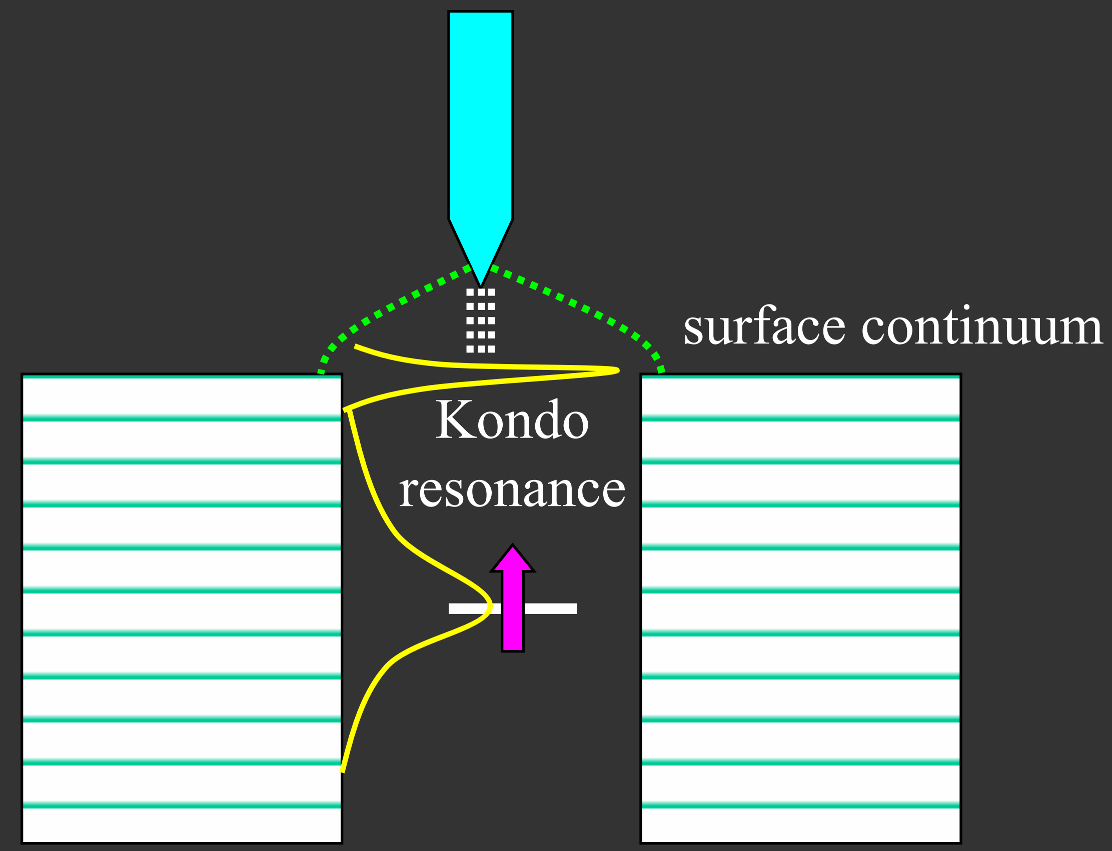


Spectra at different distances from the Co atom

- Co in Au is a known Kondo system with fairly high T_k
- A sharp spectroscopic feature around ε_F right on Co atom, but not off the atom.
- The feature is not a sharp peak expected from the Kondo resonance.
- The feature dies away at around 10 Å from the Co atom.

Fano resonance



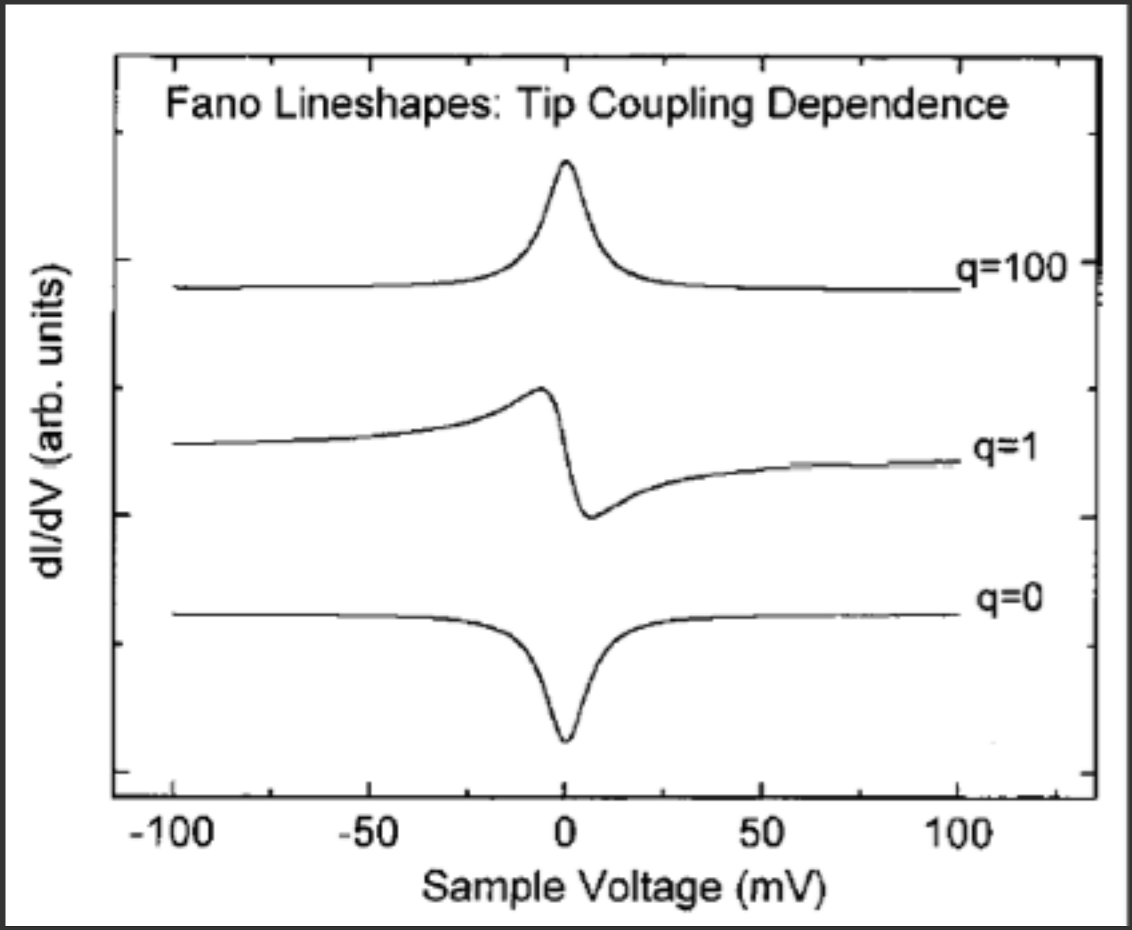


- There are two different STM tunneling channels, one through the impurity (the Kondo resonance) and one directly into the metal substrate (continuum states).
- When there is interference between the discrete and continuum tunneling channels, the tunneling spectra will be modified from a Lorentzian line-shape into an asymmetric Fano line-shape.
- This called the Fano Resonance (Ugo Fano 1961).

Fano resonance in STM spectrum

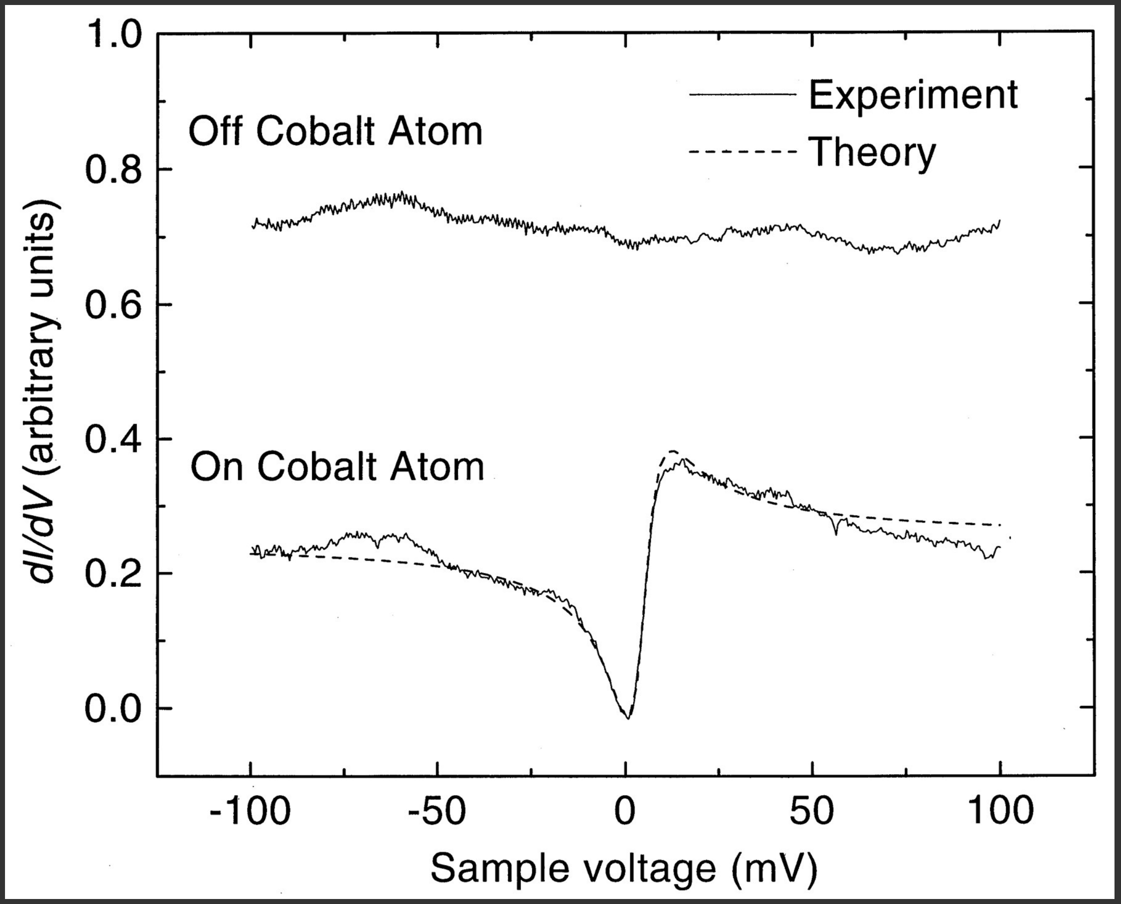
In the Fano model, the STM spectrum looks like: $$\frac{dI}{dV} \sim \frac{(q+\varepsilon')^2}{1+\varepsilon'^2}$$

q is the ratio of tunneling into the discrete and continuum states



- $q >> 1$: tunneling mainly through the impurity, so the Fano resonance is close to the Lorentzian, symmetric Kondo resonance peak.
- $q << 1$, tunneling mainly through the continuum, whose DOS is depleted by the impurity, anti-resonance.
- $q \sim 1$, tunneling is through both channels, asymmetric line-shape.

Theoretical fit to the Fano resonance

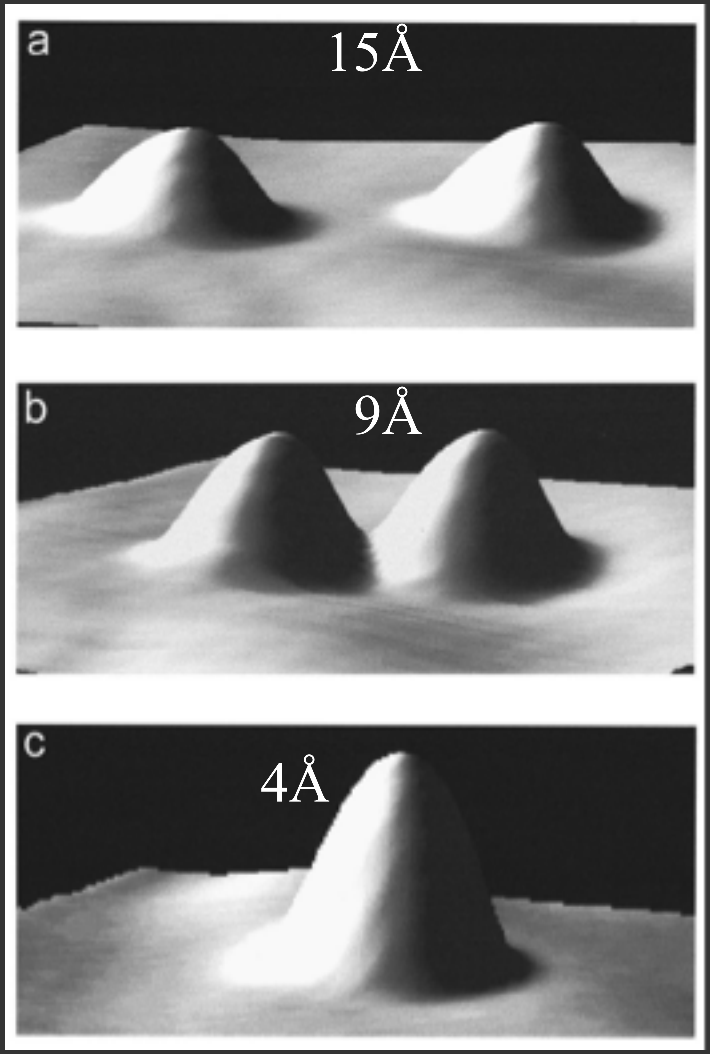


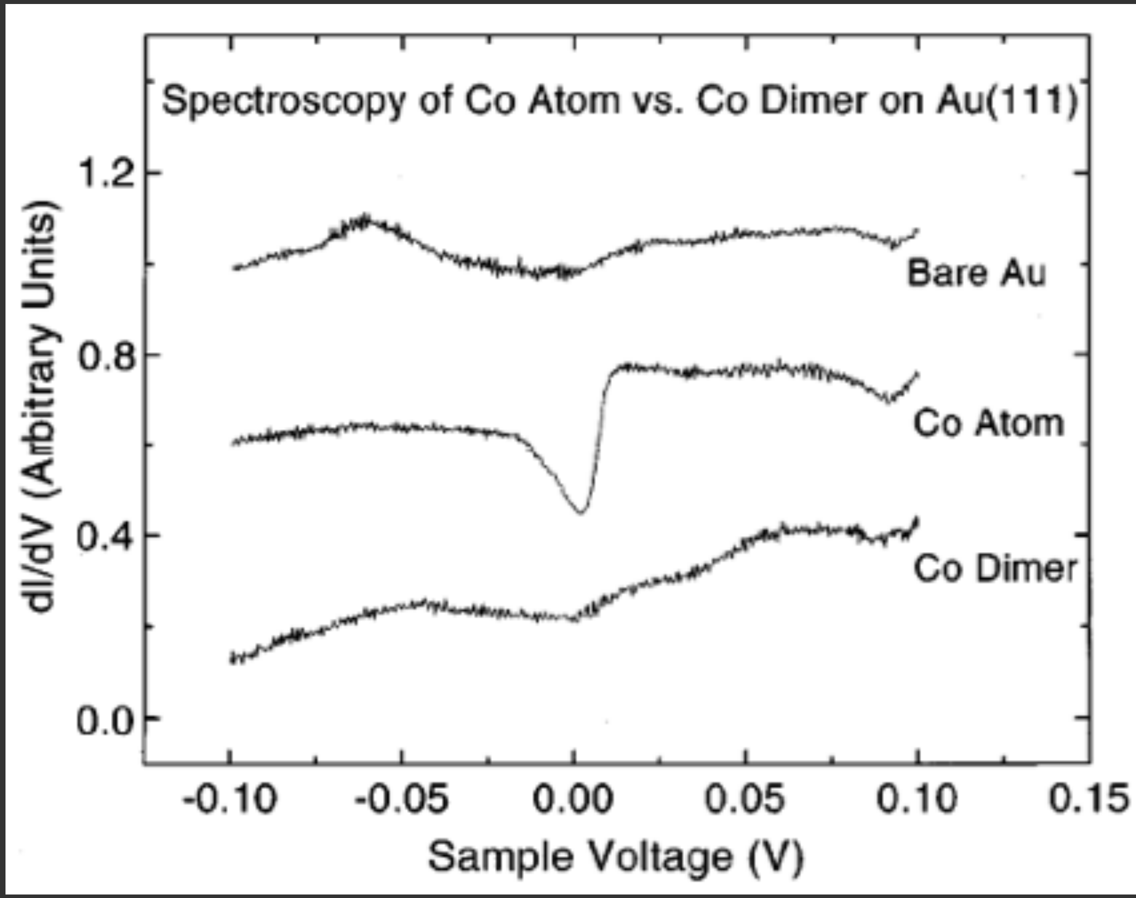
- Excellent fit using the Fano model. The parameters are $k_B T_K = 5.5$ meV, $\varepsilon_0 = 4.5$ meV, and $q = 0.7$.
- There is a fairly strong mixing between the two channels.
- The T_k for a Co atom on Au(111) is 70 K, so the measurements were made in the $T < T_K$ limit.

- The T_K value is much lower than values for Co impurities in bulk Au, which range from 300 to 700 K.

- The lower surface T_K is because Co impurities in the bulk have more neighboring atoms, thus larger overlap of the *d* orbital with conduction electrons, and thus wider Kondo resonance.

Kondo effect in impurity dimer



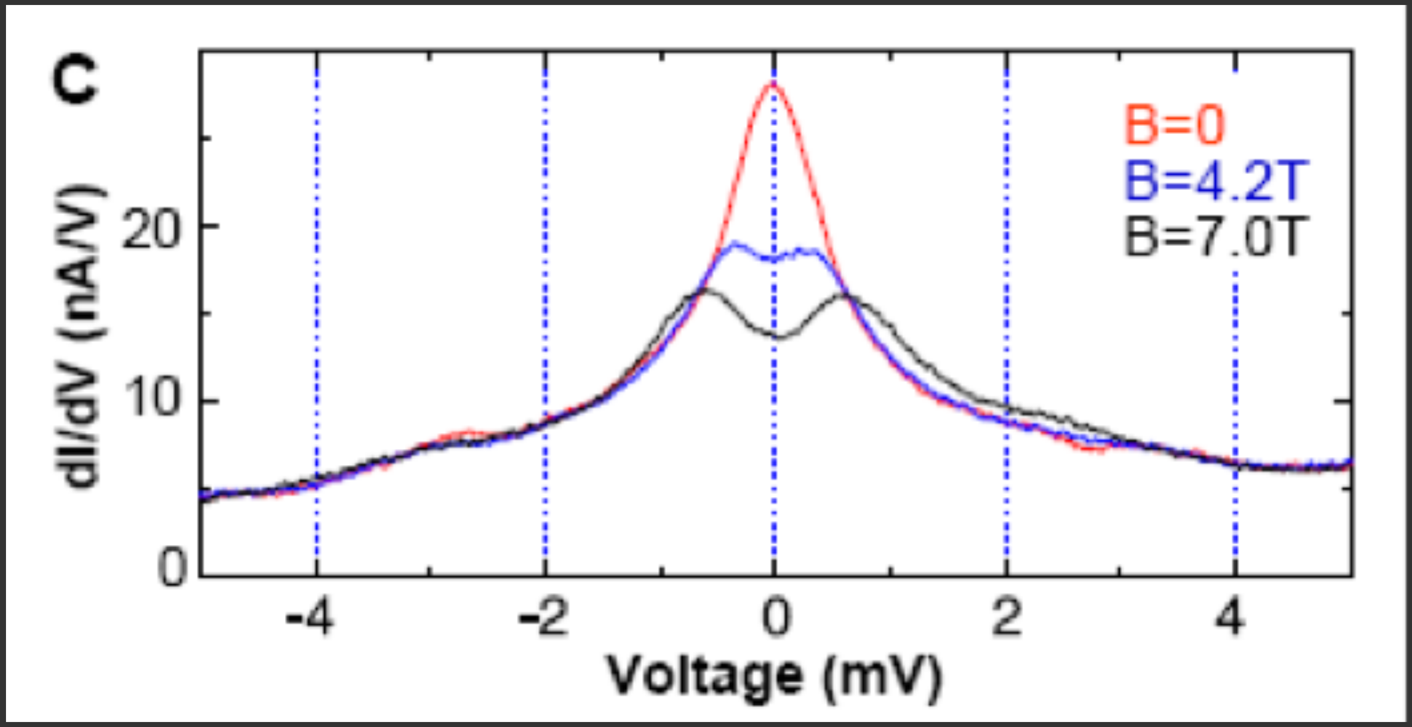


Kondo resonance disappears for a Co dimer
Crommie group (1999)

- The Kondo effect disappears for Co dimer (two Co atoms that are in close proximity to each other)
- What are the possible causes?

Effect of magnetic field on Kondo resonance

Mn atoms on Al_2O_3/NiAl
Heinrich et al, (2004)

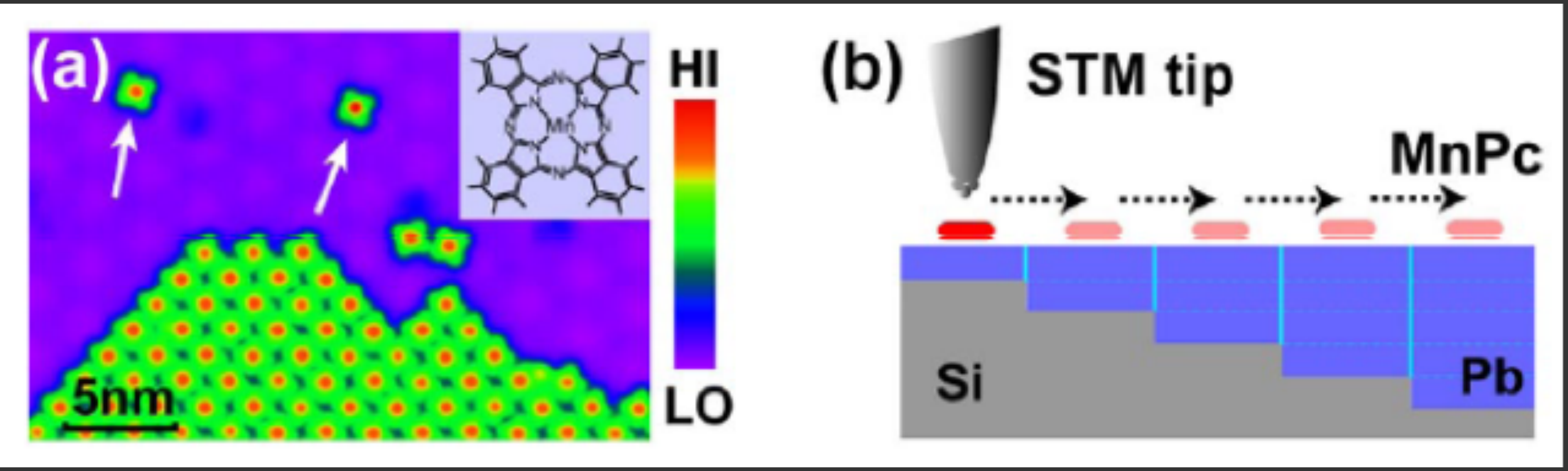


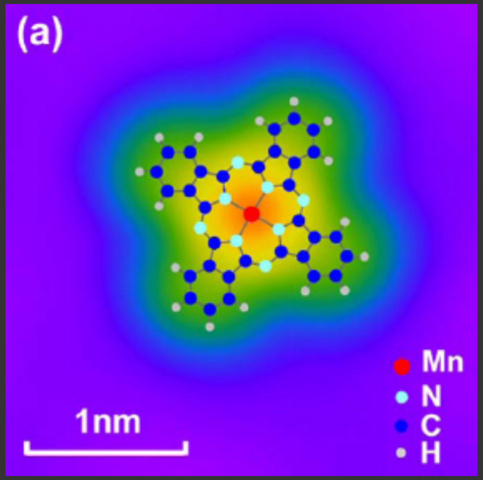
Kondo resonance on Mn atoms in a magnetic field

- At zero field, there is a sharp Lorentzian Kondo resonance peak.
- $T_K \sim 6$ K from the resonance width, data were taken at $T = 0.6$ K $< T_K$.
- Magnetic fields split the Kondo peak, the splitting is proportional to the field.
- Zeeman splitting of the impurity spin energy levels.

Kondo effect of single magnetic molecule

- Molecular magnet may have novel tunable magnetic properties due to exchange coupling via organic ligands.
- They can also be used in molecular spintronics devices, which may achieve the ultimate miniaturization of magnetic storage and computation.
- Kondo effect has been seen in single molecule magnet.

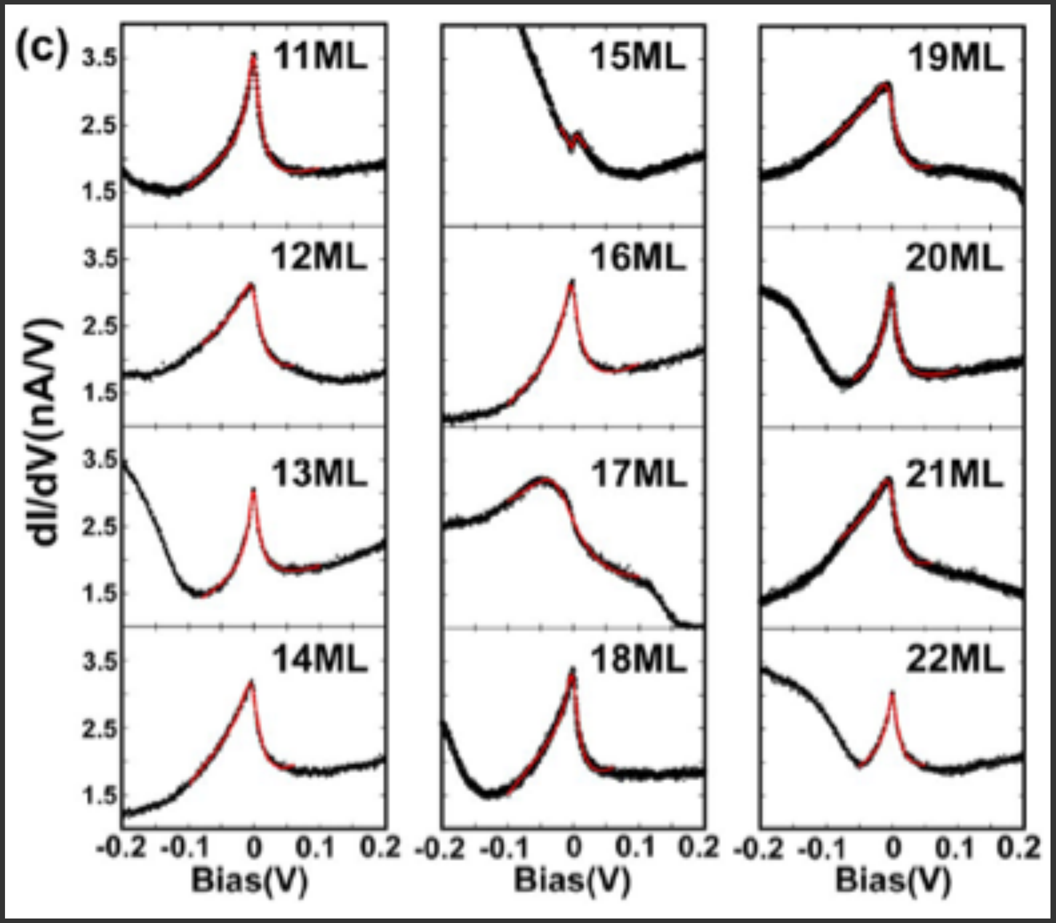


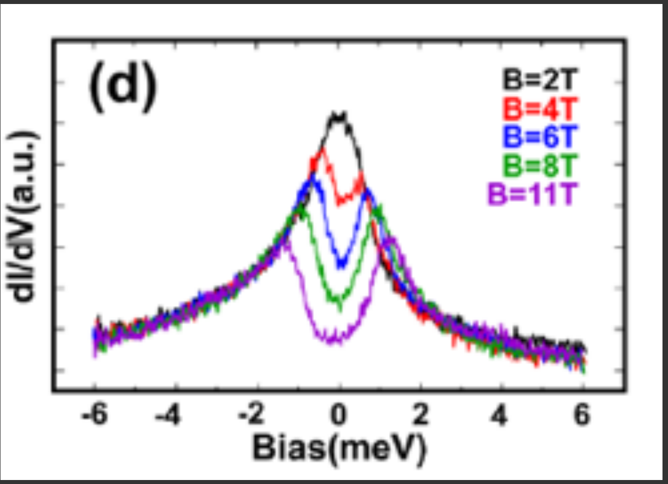


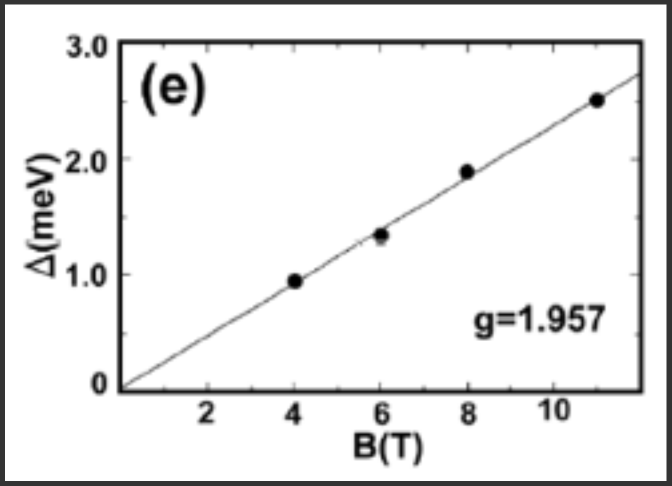
MnPc molecules on Pb quantum well on Si(111) substrate

Xue Group (2007), PRL cover story

Kondo effect of single magnetic molecule



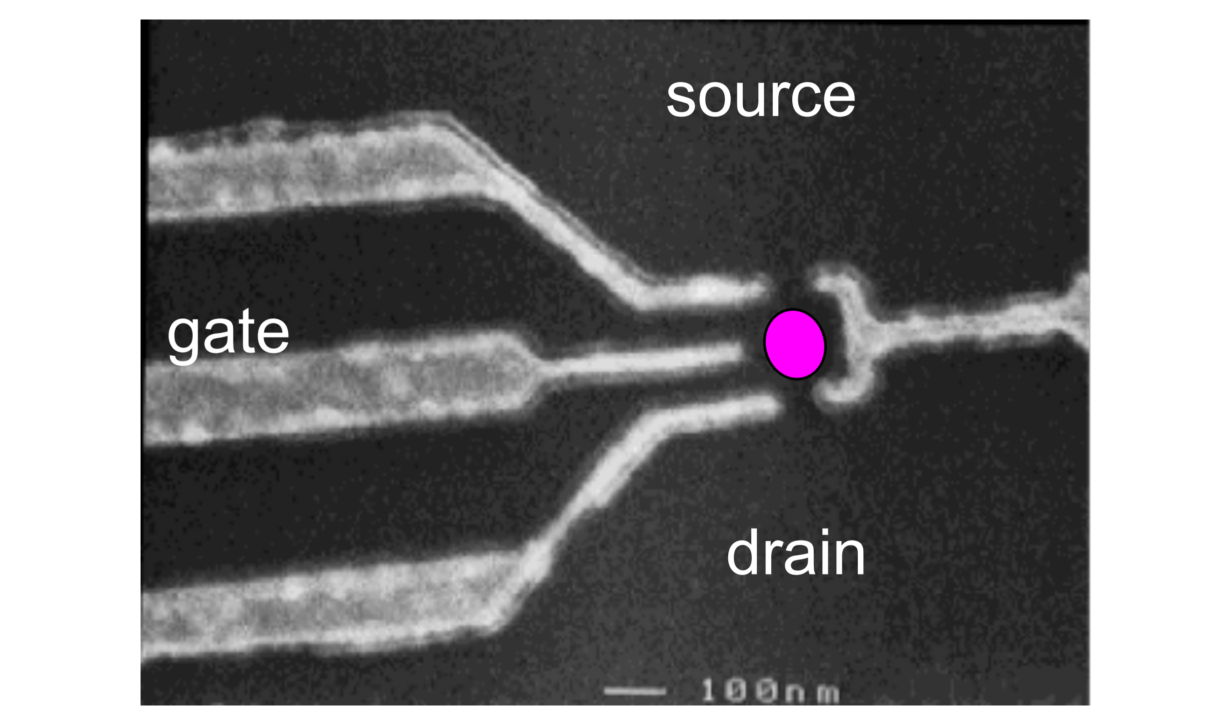




- What are the main patterns? What are the possible causes?
- The DOS of the Pb quantum well structure can be tuned precisely by varying the layer structure.
- We thus have a tunable molecular Kondo system.

Kondo effect in quantum dot

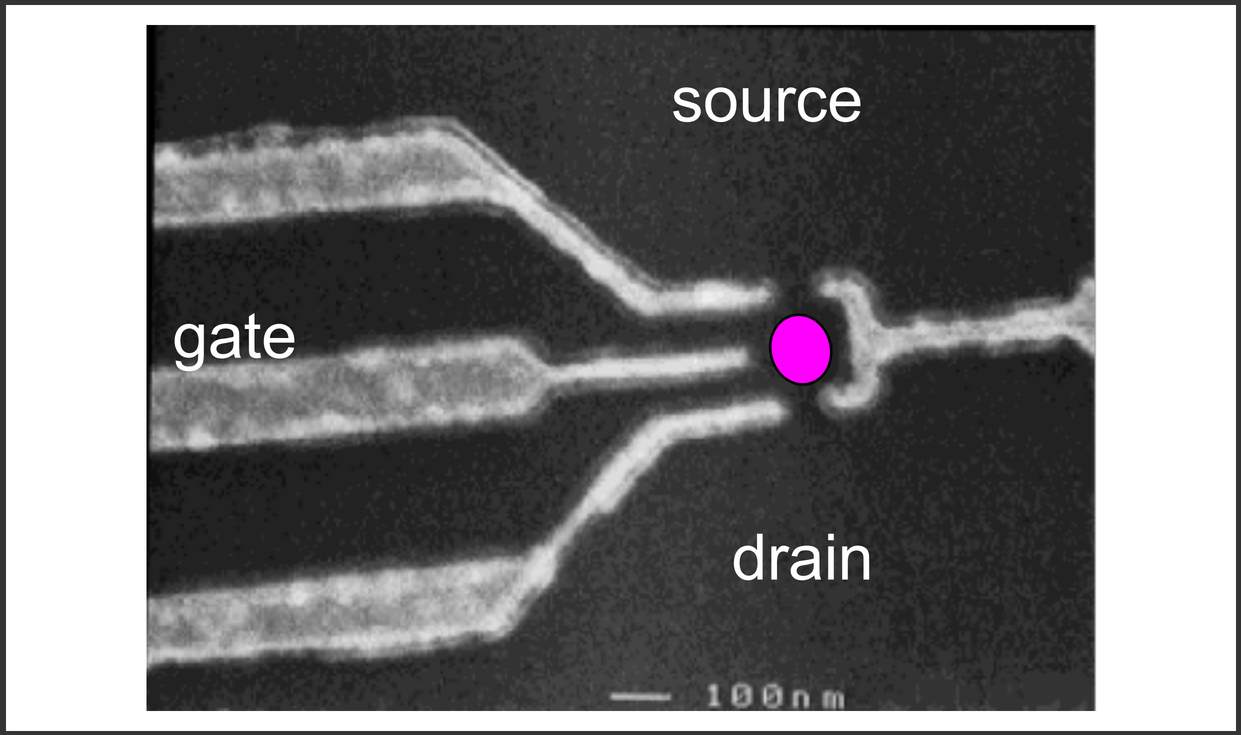
- Semiconductor QD has been proposed to be an ideal system to study the Kondo physics under controlled circumstances.



- How to create a Kondo system using a QD?
- An unpaired electron on the quantum dot can act as a magnetic impurity.
- The conducting electrons in the leads play the role of host metal.

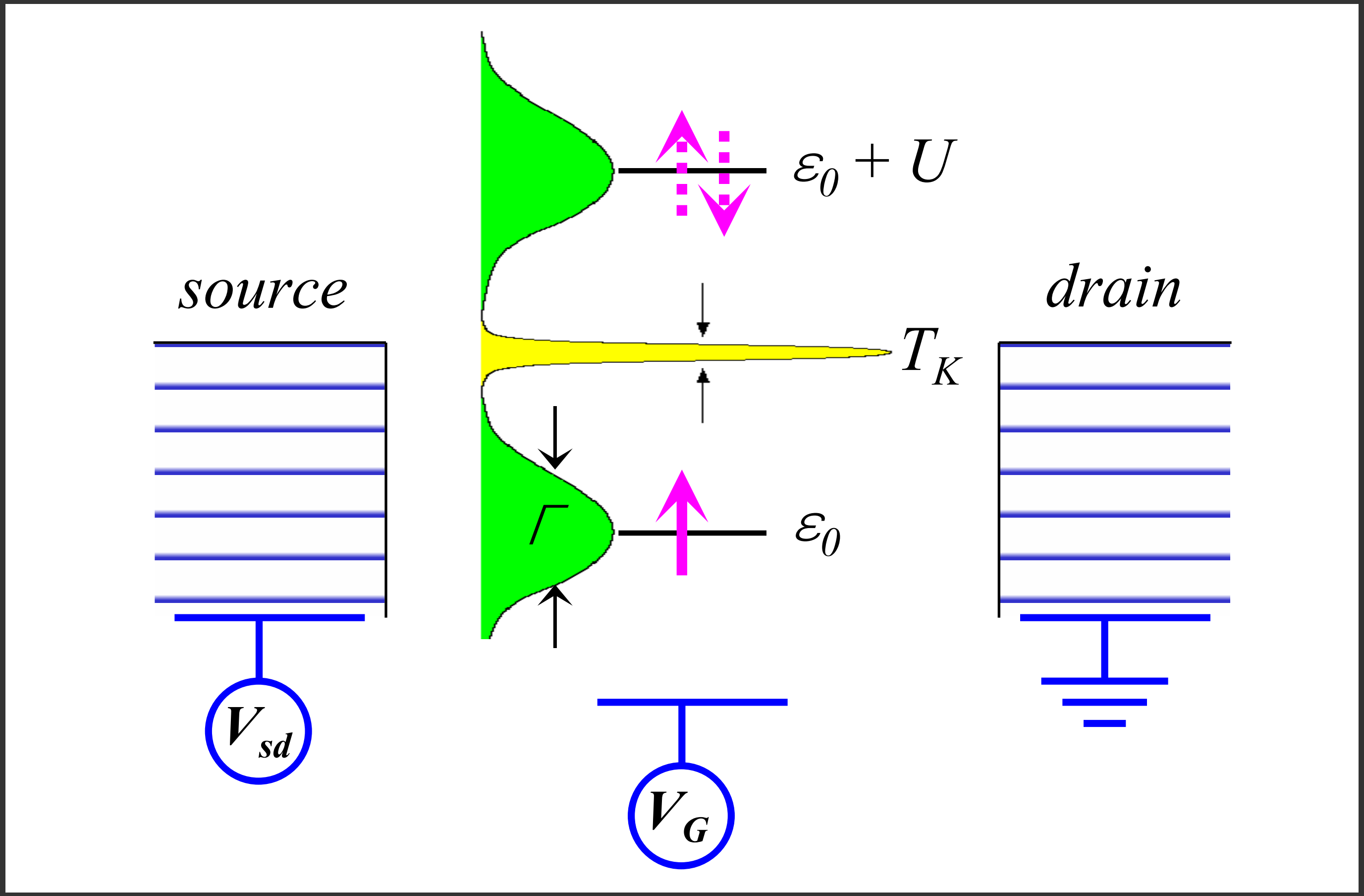
"Kondo effect in a single-electron transistor", Goldhaber-Gordon et al., Nature (1998).

Advantages of QD in studying Kondo effect



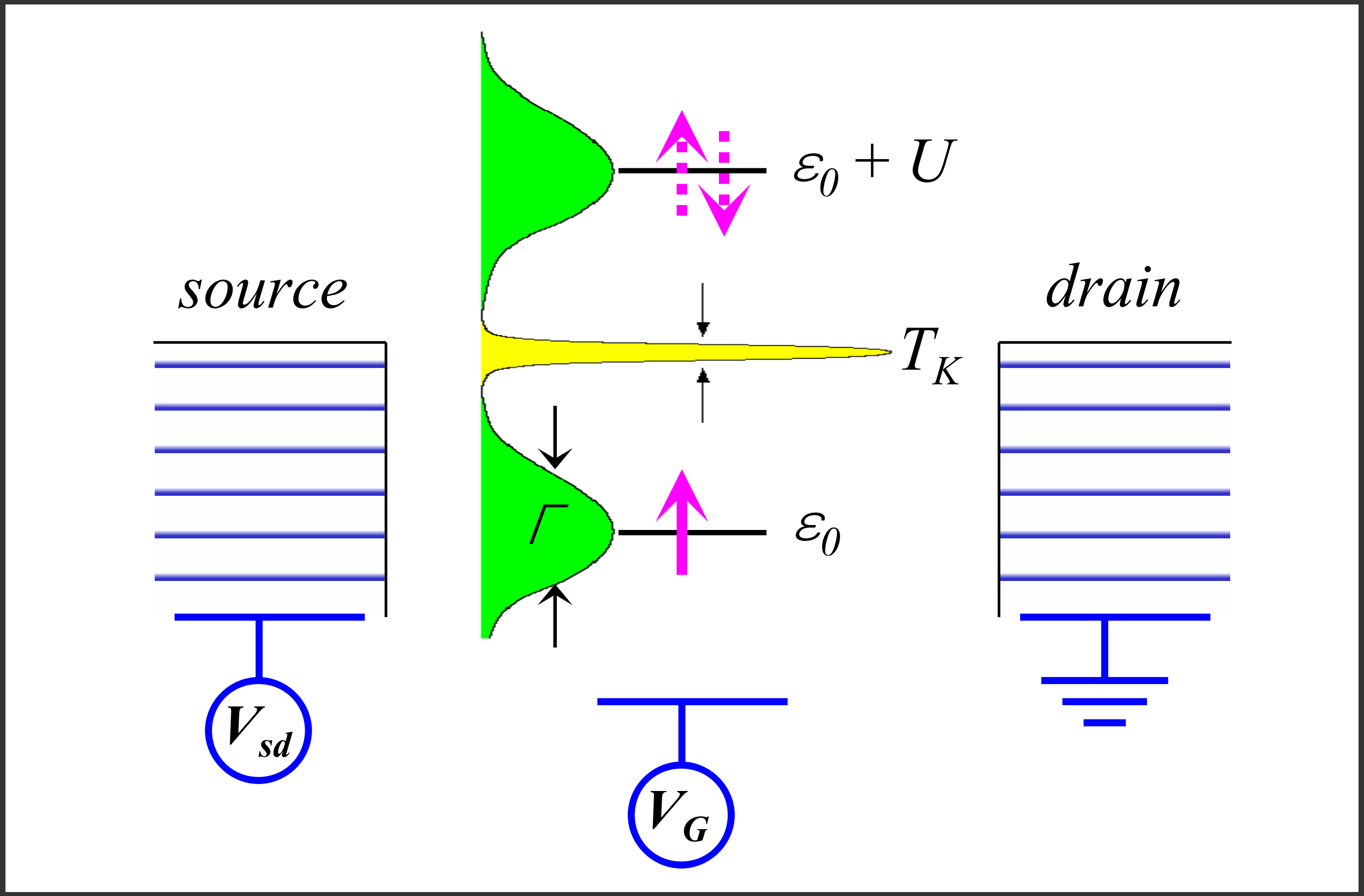
- A single localized state can be studied rather than a statistical distribution
- The number of localized electrons can be changed from odd to even
- Energy difference between the localized state and the Fermi level can be tuned
- The coupling to the conduction electrons in the leads can be adjusted
- Bias voltage can be applied to reveal non-equilibrium Kondo phenomena
- *The QD offers unprecedented control of all the relevant parameters*

Energy diagram of QD Kondo system



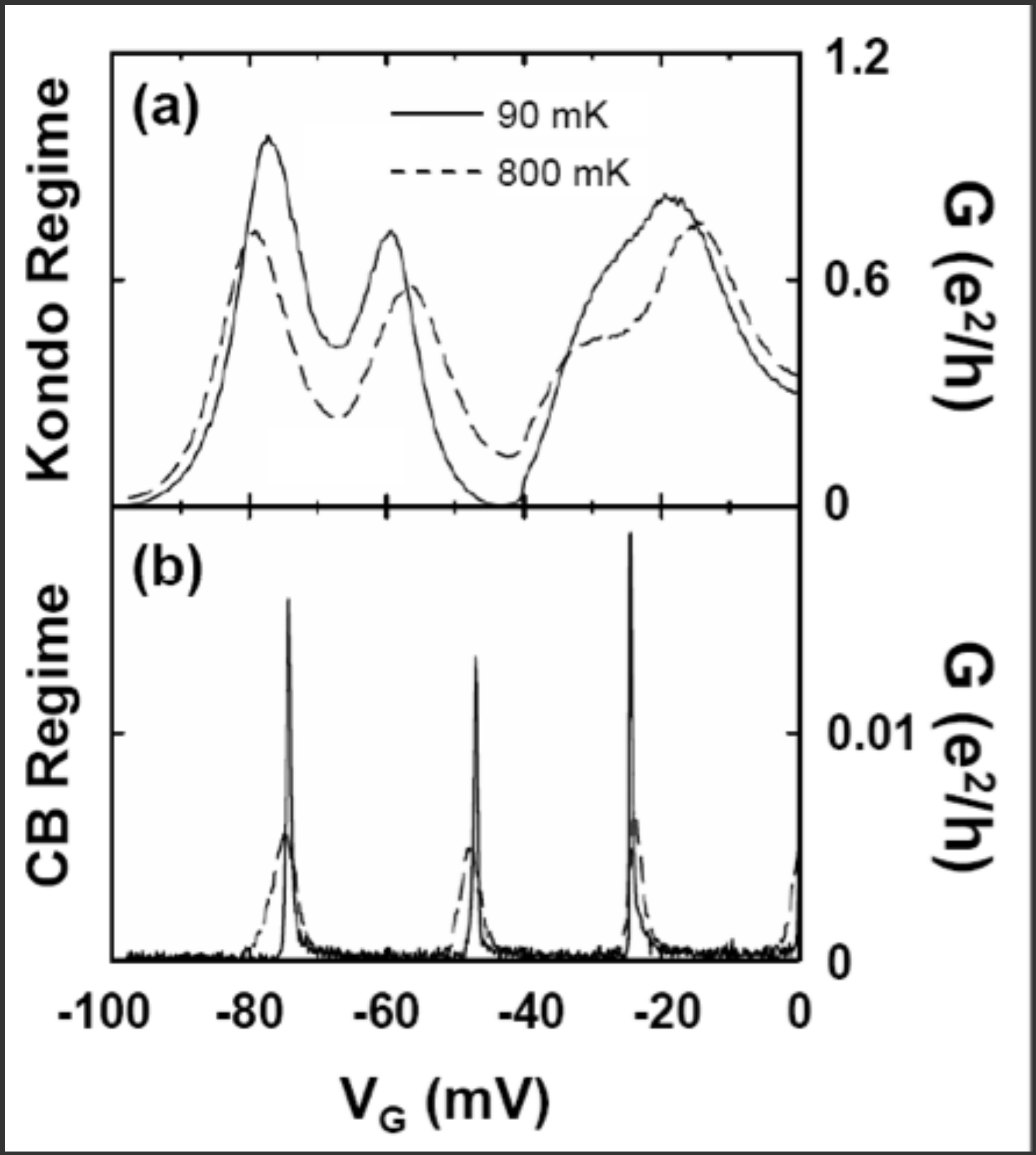
- ε_0: position of the unpaired electron on QD
- U: charging energy e^2/C caused by Coulomb repulsion on the QD
- Γ: coupling of the electrons on QD with conduction electrons on the leads
- T_K: Kondo temperature, in QD: $T_K \sim \sqrt{\Gamma U} \exp\left(\pi\varepsilon_0(\varepsilon_0 + U)/\Gamma U\right)$

Zero bias conductance peak



- What is the signature of Kondo resonance in the conductance of QD?
- When the QD is singly occupied, the conductance between source and drain will show a sharp peak due to the Kondo resonance DOS peak at E_F
- The peak width, its temperature and field dependence can reveal important information regarding the Kondo physics.

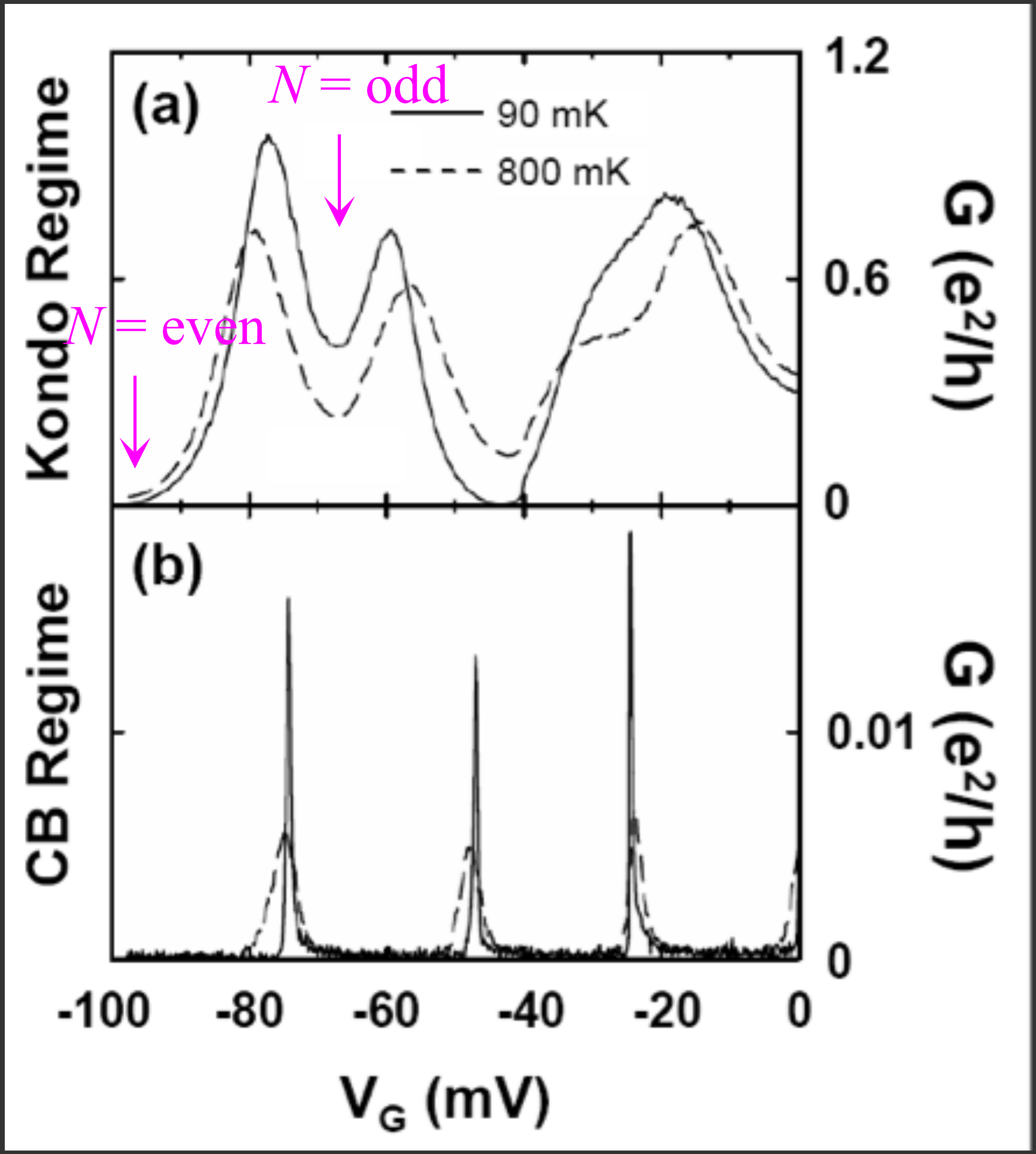
Signature of Kondo effect in QD



Goldhaber-Gordon et al., Nature (1998).

- Let's first look at the lower panel.
- This is when the coupling between the electrons on QD and the conduction electrons on leads is very weak (Γ is very small)
- What is the main feature?
- What is the cause for this?
- Is there Kondo effect?
- WHY?

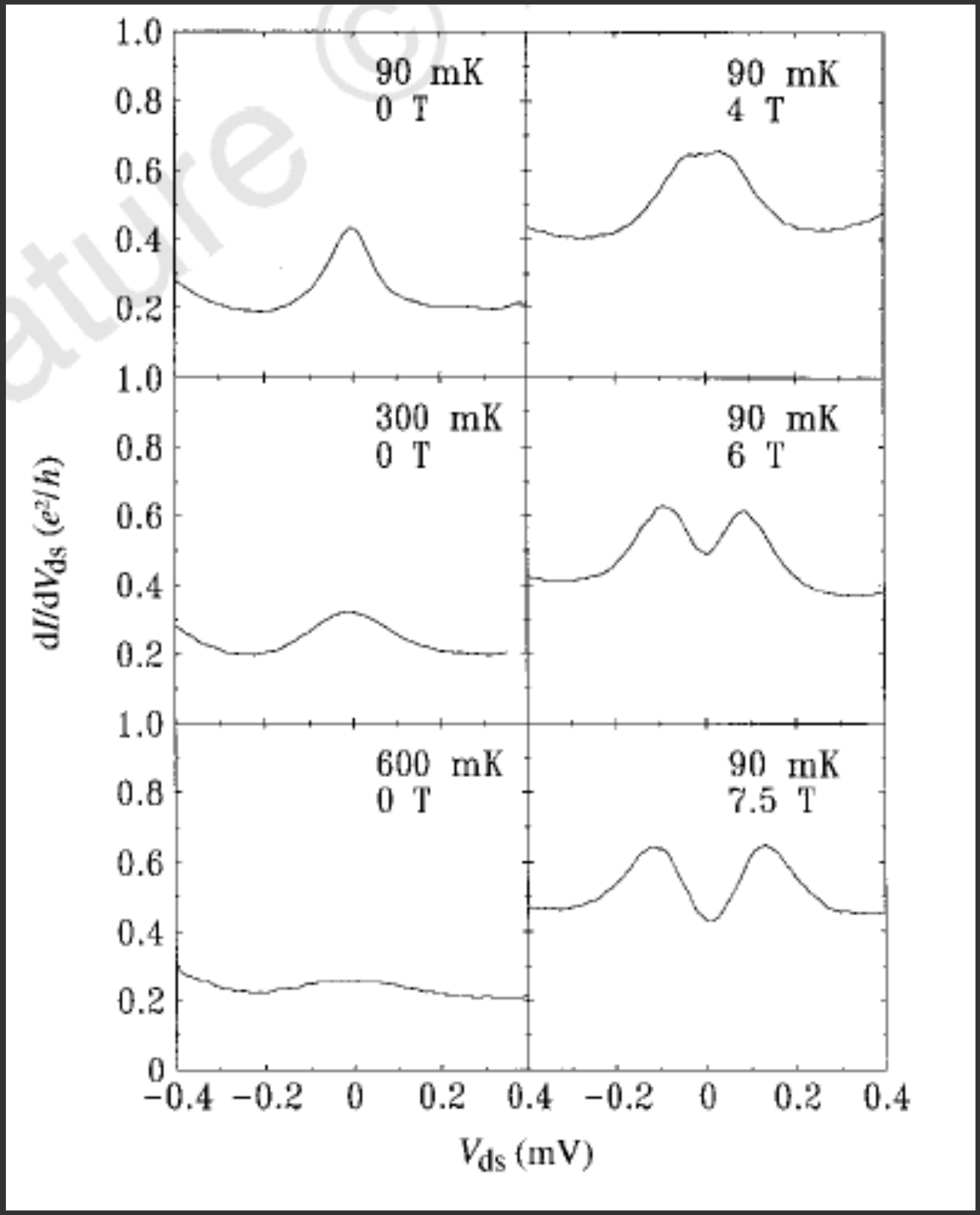
Signature of Kondo effect in QD



Goldhaber-Gordon et al., Nature (1998).

- Let's look at the upper panel.
- This is when Γ is relatively large.
- What is the new feature?
- Even-odd oscillation of the amplitude of the conductance.
- For odd number of electrons, the conductance is significantly enhanced, WHY?
- For odd number of electrons, there is Kondo effect with large T_K.
- The enhanced conductance is due to the Kondo resonance DOS peak

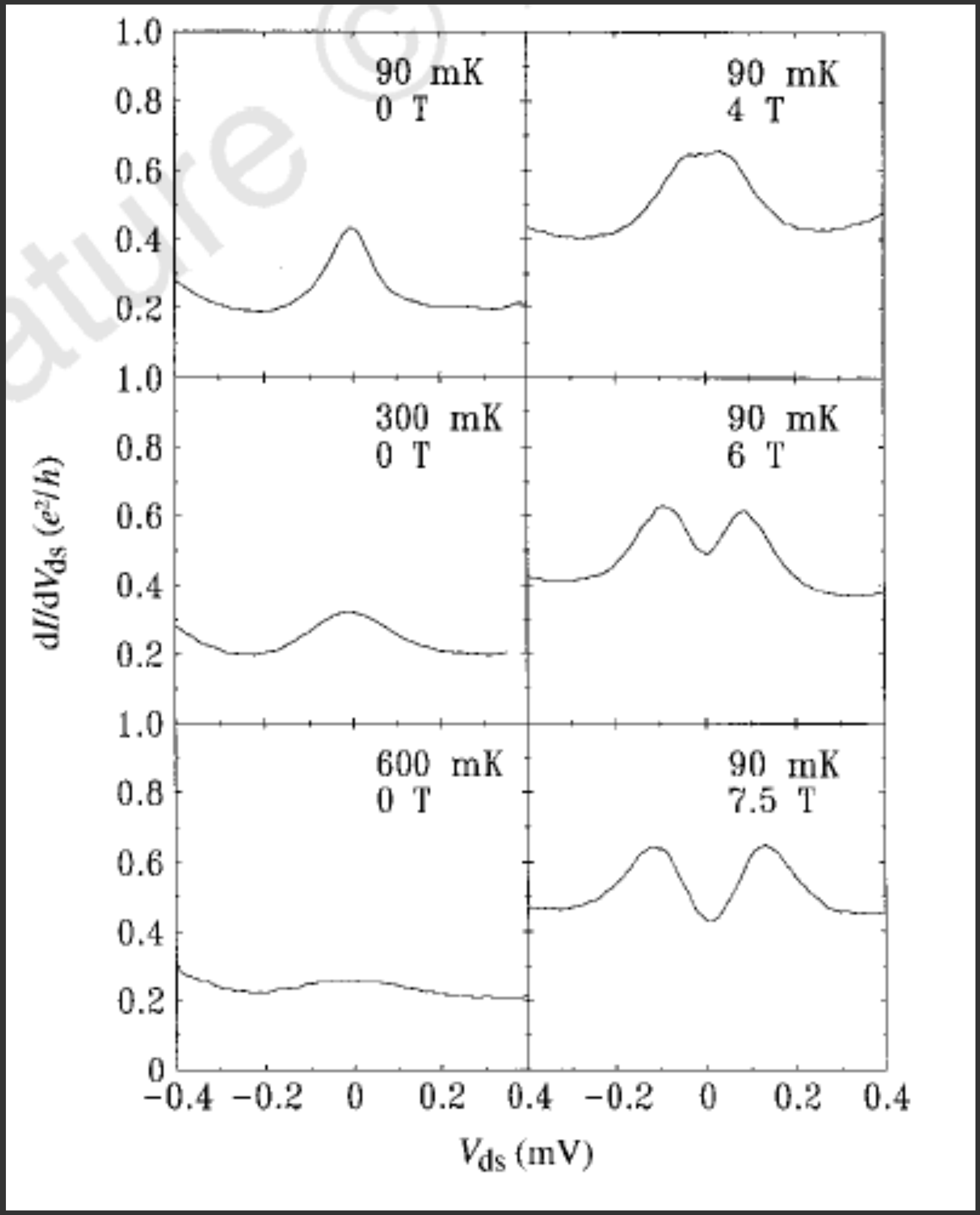
The zero bias conductance peak



Goldhaber-Gordon et al., Nature (1998).

- Left panel shows the conductance of a singly occupied QD as a function of bias voltage.
- It shows a sharp conductance peak right at zero bias.
- The peak decays rapidly with increasing bias (~ 0.1 mV)
- The peak also decays rapidly with rising temperature (at 600 mK it is almost completely gone).
- WHY?
- How high is T_K?

The zero bias conductance peak

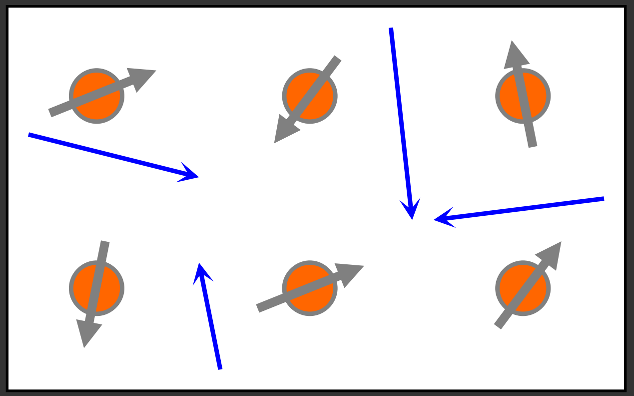


Goldhaber-Gordon et al., Nature (1998).

- Right panel shows the conductance in different magnetic field.
- The single peak at zero field splits into two peaks.
- The splitting is proportional to the magnetic field.
- WHY?
- This is due to the Zeeman splitting of the unpaired electron on the QD.

Kondo Lattice and heavy fermion

- So far we only focus on Kondo effect caused by dilute magnetic impurities without considering the interaction between the impurities themselves.
- What about dense magnetic impurities that interact with each other and with conduction electrons?



- A Kondo lattice is one of such systems with a periodic array of local moments.
- It is a valid model for the *f*-electron heavy fermion systems, where interactions with local moments leads to quasiparticles with unusually large effective mass.
- The heavy fermion systems are among the most complicated materials due to the existence of many competing interactions and phases.

References: "The Kondo Problem to Heavy Fermions" by A. C. Hewson (1995).

Summary of the Kondo Effect

- Resistivity minimum in metals with dilute magnetic impurities
- Anderson Model: local moment due to Coulomb repulsion U
- Kondo Model: the log$T$ term due to second order spin-flip scatterings when the exchange interaction is antiferromagnetic.
- Anderson and Wilson solved the Kondo divergence problem.
- Kondo effect is a quantum many-body effect due to the formation of screening cloud and Kondo resonance by conduction electrons.

Summary of the Kondo Effect

- Kondo effect is one of the major puzzles solved in CMP, comparable to the solution of BCS superconductivity.
- It is a landmark in the modern quantum many body physics.
- It has many connections with other branches of physics.
- It is the first known example of asymptotic freedom, in which the coupling becomes non-perturbatively strong at low T/low energies.
- It is still an active area of research in recent years, especially in low-dimensional electron systems and Kondo lattices.