第二次作业答案

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Ch2 9(3)

> 求所有整数解 15x + 16y = 17.

特解

$$\begin{cases} x = -1 \\ y = 2 \end{cases}$$

故所有整数解为

$$\begin{cases} x = x_0 + \dfrac{b}{(a,b)}t = -1 + 16t \\ y = y_0 - \dfrac{a}{(a,b)}t = 2 - 15t \end{cases} (t \in Z)$$

> 注意：这里$t \in Z$不能漏，b和a不要写反了

Ch2 18(2)

> 解线性同余方程 $3x \equiv 6 \pmod{18}$.

因 (3, 18) = 3, 3 | 6, 有 3 个模 18 不同余的解.

考察 x ≡ 2 (mod 6). x ≡ 2 + 6t(mod 18), 0 ≤ t ≤ 2 是所求解, 即解为 x ≡ 2, 8, 14 (mod 18)

Ch2 19(4)

> 解同余方程组：
>
> $$\begin{cases} 2x \equiv 1 \pmod{5} \\ 3x \equiv 2 \pmod{7} \\ 4x \equiv 1 \pmod{11} \end{cases}$$

方程组等价为

$$\begin{cases} x \equiv 3 \pmod{5} \\ x \equiv 3 \pmod{7} \\ x \equiv 3 \pmod{11} \end{cases}$$

$$M = 5 \times 7 \times 11 = 385, M_1 = 77, M_2 = 55, M_3 = 35$$

$$77b_1 \equiv 1 \pmod{5} \Rightarrow b_1 = 3$$

$$55b_2 \equiv 1 \pmod{7} \Rightarrow b_2 = 6$$

$$35b_3 \equiv 1 \pmod{11} \Rightarrow b_3 = 6$$

$$\Rightarrow 77 \times 3 \times 3 + 55 \times 6 \times 3 + 35 \times 6 \times 3 \equiv 2313 \equiv 3 \pmod{385}$$

> 注意：步骤过程按照书上例题来

Ch2 22

> 计算 $\phi(42), \phi(420), \phi(4200)$.

分别将42，420，4200分解质因数，代入公式计算即可

$$42 = 2 \times 3 \times 7, 420 = 2^2 \times 3 \times 5 \times 7, 4200 = 2^3 \times 3 \times 5^2 \times 7,$$

$$\phi(42) = \phi(2) * \phi(3) * \phi(7) = 1 * 2 * 6 = 12$$

$$\phi(420) = \phi(2^2) * \phi(3) * \phi(5) * \phi(7) = (2 * 1) * 2 * 4 * 6 = 96$$

$$\phi(4200) = \phi(2^3) * \phi(3) * \phi(5^2) * \phi(7) = (2^2 * 1) * 2 * (5 * 4) * 6 = 960$$

Ch2 24

> p为素数，(m,n) = p, 问$\phi(mn)$与$\phi(m)\phi(n)$之间有什么关系

设$m = p_1^{\alpha_1} * p_2^{\alpha_2} ... p_n^{\alpha_n}$
设$n = p_1^{\beta_1} * p_2^{\beta_2} ... p_n^{\beta_n}$，其中$p_i$为素数，$\alpha_i, \beta_i \geq 0$

$(m, n) = p = p_1^{\min(\alpha_1,\beta_1)} * p_2^{\min(\alpha_2,\beta_2)} ... p_n^{\min(\alpha_n,\beta_n)}$

不妨另$i = l$时，$\min(\alpha_l, \beta_l) = 1$,其他时刻$\min(\alpha_i, \beta_i) = 0$

由于p为素数，$p_l = p$,有

$$\phi(mn) = mn(1 - \frac{1}{p_1}) ... (1 - \frac{1}{p_n})$$

$$\phi(m)\phi(n) = mn(1 - \frac{1}{p_1})(1 - \frac{1}{p_2}) ... (1 - \frac{1}{p_l})^2 ... (1 - \frac{1}{p_n})$$

即可得

$$\phi(mn) = \frac{p}{p-1}\phi(m) * \phi(n)$$

Ch2 27

> 314^{159}除以7的余数是多少?

方法一:

$$314 \equiv -1 \pmod 7$$

$$314^{159} \equiv (-1)^{159} \equiv 6 \pmod 7$$

方法二:

即解$314^{159} \equiv x \pmod 7$

由Euler定理，$314^6 \equiv 1 \pmod 7$

$314^{6*26+3} \equiv x \pmod 7$

$314^3 \equiv x \pmod 7$

$(44 * 7 + 6)^3 \equiv x \pmod 7$

$6^3 \equiv x \pmod 7$

$x \equiv 6 \pmod 7$

余数为6