S As a function of q and t.

$$\bar{\delta} S = \left[\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}^i}\right) - \frac{\partial L}{\partial q^i}\right]\delta q^i + \left[p_i \bar{\delta} q^i - H \delta t\right]\Big|_{t_1}^{t_2}.$$

Now we consider those paths that ~~B~~ one of true motions, hence:

$$\bar{\delta} S = p_i \bar{\delta} q^i - H \delta t \Big|_{t_1}^{t_2}. \quad (*)$$

which has clear image that is exhibited below. By introducing this formular we aims to show that ~~not all H can represent the status of a system. because $p_i dq^i - H dt$ not all~~ the final state of the system cannot be random, it has to satisfy $(*)$.

(2) (2') (1) (1)' This provides us a method to determine the motion of the system as S have to satisfy $(*)$ if the motion is real.

We may fix our initial condition, hence ~~the~~ $(*)$ can be written as ~~one~~

$$dS = p_i dq^i - H dt \quad (1)$$

where ~~this~~ dq^i and dt here changes at the end point.

$$\Rightarrow \left(\frac{\partial S}{\partial t}\right)_q = -H, \quad \left(\frac{\partial S}{\partial q^i}\right)_{t, q^{j}} = p_i \quad (2)$$

as H is the function of q ~~and~~, p, and t, hence:

$$H(p, q, t) + \left(\frac{\partial S}{\partial t}\right)_q = 0 \Rightarrow H\left[\left(\frac{\partial S}{\partial q}\right), q, t\right] + \left(\frac{\partial S}{\partial t}\right)_q = 0 \quad (3)$$

If we have solved this function S, we may take the solution into: $\dot{q}^i = \left(\frac{\partial H}{\partial p_i}\right)_{q^i, p^j, t}$ and insert: $p_j^0 = \left(\frac{\partial S}{\partial q^j}\right)_{t, q^i}$, then everything is done. However, this method doesn't ~~for~~ provide any convenience, there may be a method that use this view more fully:

Let us do some trick to (2):

$$\left(\frac{\partial S}{\partial t}\right)_q = 0 - H \quad \left(\frac{\partial S}{\partial q^i}\right)_{t, q^j} = p_i \quad (2^*)$$

this looks just like a canonical transformation!

1 2 3 4 5 6 7 8 9 10 11 12 The month

DATE Mon Tue Wed Thu Fri Sat Sun

So we may write: $S \equiv S(q, P, t)$

$\left(\frac{\partial S}{\partial q^i}\right)_{q, P, t} = P_i$ $\quad \left(\frac{\partial S}{\partial P^i}\right)_{q, P, t} = Q^i$ $\quad \left(\frac{\partial S}{\partial t}\right)_{q, P} = K - H$ (4).

as $K \equiv 0$, then: $P_i = const$ $\quad Q^i = const$, let $P_i = \alpha_i$ $\quad Q^i = \beta^i$

Notice that we have expanded the low index of partial derivitives, this is fairly reasonable as P_i and Q^i will stay the same (they are constants).

As one may see, P_i and Q^i, 2S quantities totally, are the integration of the system. (More precisely, can be choose to be the integration of the system).

Hence we may develop a process:

(1) Solve eqs (3), there are S+1 integration constants, one of them is due to the shift of S, which is meaningless and we just throw it away.

(2) ~~Then~~ We may let these S constants to be P_is and from (4) we may solves Q^is which are also constants.

(3) Do ~~internal~~ inverse, we may get the EoMs of q^i with 2S constants P_is and Q^is (α_i and β^is)