HW11

1. (a) $I_x = \frac{1}{2\pi}\oint p_x\, dx = \frac{1}{2\pi}\int_0^{\tau_x} m\dot{x}^2 dt$ $\quad \tau_x$: x方向运动的周期

$I_y = \frac{1}{2\pi}\int_0^{\tau_y} m\dot{y}^2 dt$

$\pi\left(\frac{I_x}{\tau_x}+\frac{I_y}{\tau_y}\right) = \langle T\rangle$ 又 $\langle T\rangle = \langle V\rangle$ (Virial定理)

$E = \langle T\rangle + \langle V\rangle = 2\pi\left(\frac{I_x}{\tau_x}+\frac{I_y}{\tau_y}\right)$

(b) $\dot{\phi}_x = \frac{\partial E}{\partial I_x} = \frac{2\pi}{\tau_x} = \omega_x \qquad \phi_x = \omega_x t + \phi_{x0}$

$\dot{\phi}_y = \frac{\partial E}{\partial I_y} = \omega_y \qquad \phi_y = \omega_y t + \phi_{y0}$

简谐运动 $x = \sqrt{\frac{2I_x}{m}\cdot\frac{1}{\omega_x}}\cos\phi_x = \sqrt{\frac{2I_x}{m}\frac{1}{\omega_x}}\cos(\omega_x t+\phi_{x0})$

$y = \sqrt{\frac{2I_y}{m}\frac{1}{\omega_y}}\cos\phi_y = \sqrt{\frac{2I_y}{m}\frac{1}{\omega_y}}\cos(\omega_y t+\phi_{y0})$

这是一个李萨如图形

2. $I_r = \frac{1}{2\pi}\oint\sqrt{2mE+\frac{2mk}{r}-\frac{2mh}{r^2}-\frac{L^2}{r^2}}\,dr =$

$I_\theta = L - I_\varphi \qquad I_\varphi = L_z$

将 $L = I_\theta + I_\varphi$ 代入 I_r 表达式可得 $E(I_r, I_\theta, I_\varphi)$ 关系.

$\omega_r = \frac{\partial E}{\partial I_r}$.

$\omega_\theta = \frac{\partial E}{\partial I_\theta}$

$\omega_\varphi = \frac{\partial E}{\partial I_\varphi}$

周期运动: $n_r\omega_r = n_\theta\omega_\theta = n_\varphi\omega_\varphi$

3. H守恒 $\frac{1}{2}ml^2\dot{\theta}^2+\frac{1}{2}mgl\theta^2=\frac{1}{2}mgl\theta_0^2$ ① θ_0为振幅

$$\frac{1}{2}\frac{p^2}{ml^2}+\frac{1}{2}mgl\theta^2=\frac{1}{2}mgl\theta_0^2$$

容易求出一个周期内，摆受力的平均

$$\bar{F}=-\frac{1}{2}mg\overline{\theta^2}+ml^2\overline{\dot{\theta}^2}=-\frac{1}{4}mg\theta_0^2+\frac{1}{2}ml^2\omega^2\theta_0^2=\frac{1}{4}mg\theta_0^2$$

$$-\bar{F}dl=dH=\frac{1}{2}mg\theta_0^2dl+\frac{1}{2}mgl\cdot 2\theta_0 d\theta_0$$

$\Rightarrow -\frac{3}{4}\theta_0 dl=l d\theta_0 \quad \Rightarrow \delta(l^{\frac{3}{2}}\theta_0^2)=0$ $E\sqrt{l}$ 是绝热不变量

(由①式可知其与相空间中的面积相关)

4. $\delta E l^2+2El\delta l=0$ 即 $\delta E=-\frac{2E}{l}\delta l$