时间序列期末答案

September 2020

一、 填空题

1. $ARIMA(0,1,1)$
2. 4,$ARIMA(0,1,1)\times(1,1,1)_4$
3. $\left[\hat{Y_1}-0.098\hat{Y_0},\hat{Y_1}+0.098\hat{Y_0}\right]$
4. $2\cos\omega X_3-X_2$
5. 5.75
6. $\alpha_0+\alpha_1\epsilon_{t-1}^2$
7. $\frac{8}{21},\frac{1}{21}$
8. 用$\{X_t,t\le n\}$对X_{n+1}做线性预测的误差非0,$\lim_{k\to\infty}\sigma_k=\gamma_0$
9. $(1+\theta_1^2+\cdots+\theta_{l-1}^2)\sigma^2$
10. 0

二、

$$
\begin{aligned}
X_t &= \frac{1+0.8\mathscr{B}}{1-0.4\mathscr{B}}\epsilon_t \\
&= (1+0.8\mathscr{B})\sum_{i=0}^{\infty}0.4^i\mathscr{B}^i\epsilon_t \\
&= \epsilon_t + 3\sum_{i=1}^{\infty}0.4^i\epsilon_{t-i}
\end{aligned}
$$

故有$Var(X_t) = \left[1+9\sum_{i=1}^{\infty}0.4^{2i}\right]\sigma^2 = \frac{19}{7}\sigma^2$

$$
Cov(X_{t+k},\epsilon_t) = \begin{cases} \sigma^2 & k=0 \\ 3\times 0.4^k\sigma^2 & k>0 \\ 0 & k<0 \end{cases}
$$

$Var(\epsilon_t) = \sigma^2$,故

$$
\rho_{X,\epsilon}(k) = \begin{cases} 0 & k<0 \\ \sqrt{\dfrac{7}{19}} & k=0 \\ \sqrt{\dfrac{7}{19}}\times 3\times 0.4^k & k>0 \end{cases}
$$

三、

讨论，$l=2k$时,$\hat{X}_t(l) = \frac{1}{9^k}X_t$, $l=2k-1$时, $\hat{X}_t l = \frac{1}{9^k}X_{t-1}$.
对应的预测方差为$\mathrm{Var}(\hat{X}_t(l)-X_{t+l}) = (1+\frac{1}{9^{2k}})\gamma_0 - \frac{2}{9^k}\gamma_{2k}$.
由此易知预测方差趋于γ_0.

四、

参见2019年期末试卷，答案为0.2836,1.018,是正态分布

五、

1、

$$(1-\phi_1\mathscr{B})X_t = \phi_0 + (1-\theta_1\mathscr{B})\epsilon_t$$

$$\begin{aligned}
X_t &= \frac{\phi_0}{1-\phi_1\mathscr{B}} + \frac{1-\theta_1\mathscr{B}}{1-\phi_1\mathscr{B}}\epsilon_t \\
&= \frac{\phi_0}{1-\phi_1} + (1-\theta_1\mathscr{B})\sum_{i=0}^{\infty}\phi_1^i\mathscr{B}^i\epsilon_t \\
&= \frac{\phi_0}{1-\phi_1} + (1-\theta_1\mathscr{B})\sum_{i=0}^{\infty}\phi_1^i\epsilon_{t-i} \\
&= \frac{\phi_0}{1-\phi_1} + \sum_{i=1}^{\infty}\phi_1^i\epsilon_{t-i} - \theta_1\sum_{i=1}^{\infty}\phi_1^i\epsilon_{t-i-1} \\
&= \frac{\phi_0}{1-\phi_1} + \epsilon_t + (1-\frac{\theta_1}{\phi_1})\sum_{i=1}^{\infty}\phi_1^i\epsilon_{t-i}
\end{aligned}$$

2、

$$\begin{aligned}
\epsilon_t &= \frac{1-\phi_1\mathscr{B}}{1-\theta_1\mathscr{B}}X_t - \frac{\phi_0}{1-\theta_1\mathscr{B}} \\
&= -\frac{\phi_0}{1-\theta_1} + X_t + (1-\frac{\phi_1}{\theta_1})\sum_{i=1}^{\infty}\theta_1^i X_{t-i}
\end{aligned}$$

3、

对1.中结果求期望，得$E(X_t) = \frac{\phi_0}{1-\phi_1}$
$k=0$时：

$$\begin{aligned}
Cov(X_t, X_t) &= \left[1 + (1-\frac{\theta_1}{\phi_1})^2\frac{\phi_1^2}{1-\phi_1^2}\right]\sigma^2 \\
&= \left[1 + \frac{(\phi_1-\theta_1)^2}{1-\phi_1^2}\right]\sigma^2
\end{aligned}$$

$k > 0$时：

$$
\begin{aligned}
Cov(X_t, X_{t+k}) &= \left[(1 - \frac{\theta_1}{\phi_1})\phi_1^k + (1 - \frac{\theta_1}{\phi_1})^2 \frac{\theta_1^{k+2}}{1 - \phi_1^2}\right] \sigma^2 \\
&= \left[(1 - \frac{\theta_1}{\phi_1}) + \frac{(\theta_1 - \phi_1)^2}{1 - \phi_1^2}\right] \phi_1^k \sigma^2
\end{aligned}
$$