1. We konw that

$$
\begin{aligned}
A(z)^{-1} &= \frac{1}{(1-z/z_1)(1-z/z_2)\cdots(1-z/z_p)} \\
&= \frac{c_1}{1-z/z_1} + \frac{c_2}{1-z/z_1} + \cdots + \frac{c_p}{1-z/z_p}.
\end{aligned}
$$

   By dividing the equation, we have $\sum_{j=1}^{p}\left(c_j \prod_{i\neq j}(1-z/z_i)\right) = 1$. Let $z = z_j$ $(j = 1,\ldots,p)$, we have $c_j \prod_{i\neq j}(1-z/z_i) = 1$. Namely,

$$
c_j = \prod_{i\neq j} \frac{z_i}{z - z_i}, \quad j = 1,\ldots,p.
$$

2. Assume that $A(L)X_t = \epsilon_t$ and $B(L)Y_t = \eta_t$, where $A(z) = 1 - \sum_{i=1}^{p} a_i z^i$ and $B(z) = 1 - \sum_{i=1}^{p} b_i z^i$, respectively.
   Then the sufficient condition is $a_i = b_i$ for $i = 1,\ldots,p$. The proof is simple, thus omit.

   Or $\mu_x\mu_y = 0$ and Γ_{p+1} be positive defined.

3. By Yule-Walker equation, it is easy to calculate that

$$
(\rho_1,\ldots,\rho_5) = (-0.357, 0.756, -0.333, 0.577, -0.297).
$$

4. By the condition, $\gamma_Y(k) = \sum_{j=-\infty}^{+\infty}\sum_{i=-\infty}^{+\infty} \gamma_j\gamma_i\gamma_{j-i+k}$. Then

$$
f_Y(\lambda) = \frac{1}{2\pi}\sum_{k=-\infty}^{+\infty}\gamma_Y(k)\mathrm{e}^{-\mathrm{i}k} = 4\pi^2 f_X^3(\lambda).
$$

   By the conclusion 3.2 in chapter 2, we can demonstrate Y is a AR($3p$) series.

5. Assume that $\eta_{t-1} = -\phi^{-1}\epsilon_t$, then $X_{t-1} = \phi^{-1}X_t + \eta_{t-1}$. Namely, $X_t = \sum_{j=0}^{\infty}\phi^{-j}\eta_{t+j}$.
   Then

$$
\begin{aligned}
Z_t &= X_t - \phi^{-1}X_{t-1} \\
&= (1-\phi^{-2})\sum_{j=0}^{\infty}\phi^{-j}\eta_{t+j} - \phi^{-1}\eta_{t-1} \\
&= (\phi^{-2}-1)\sum_{j=1}^{\infty}\phi^{-j}\epsilon_{t+j} + \phi^{-2}\epsilon_t.
\end{aligned}
$$

   It is easy to prove that $\{Z_t\} \sim \mathrm{WN}\left(0, \dfrac{\sigma^2}{\phi^2}\right)$.