September 6, 2023

1. *Exercise E1.1 (i)–(iv)* in the textbook.

2. *Exercise E1.2* in the textbook. Note that:

   (a) The function of an Hermitian operator $\hat{A}$ is defined via the Taylor expansion, namely

   $$f(\hat{A}) \equiv \sum_{n=0}^{\infty} \frac{\hat{A}^n}{n!} f^{(n)}(0), \tag{1}$$

   with $f(x)$ being an analytic function. For example,

   $$e^{\hat{A}} = \hat{I} + \hat{A} + \frac{1}{2}\hat{A}^2 + \cdots + \frac{1}{n!}\hat{A}^n + \cdots . \tag{2}$$

   (b) The derivative operator $\frac{\partial}{\partial x}$ must act on a function $g(x)$. So the question *(ii.a)* is to prove

   $$\left[\frac{\partial^n}{\partial x^n}, x\right] g(x) = n \frac{\partial^{n-1}}{\partial x^{n-1}} g(x). \tag{3}$$

3. In this problem, you will try to solve the Schrödinger equation of a two level system. The Hamiltonian reads

   $$\hat{H} = \beta(|1\rangle\langle 2| + |2\rangle\langle 1|), \tag{4}$$

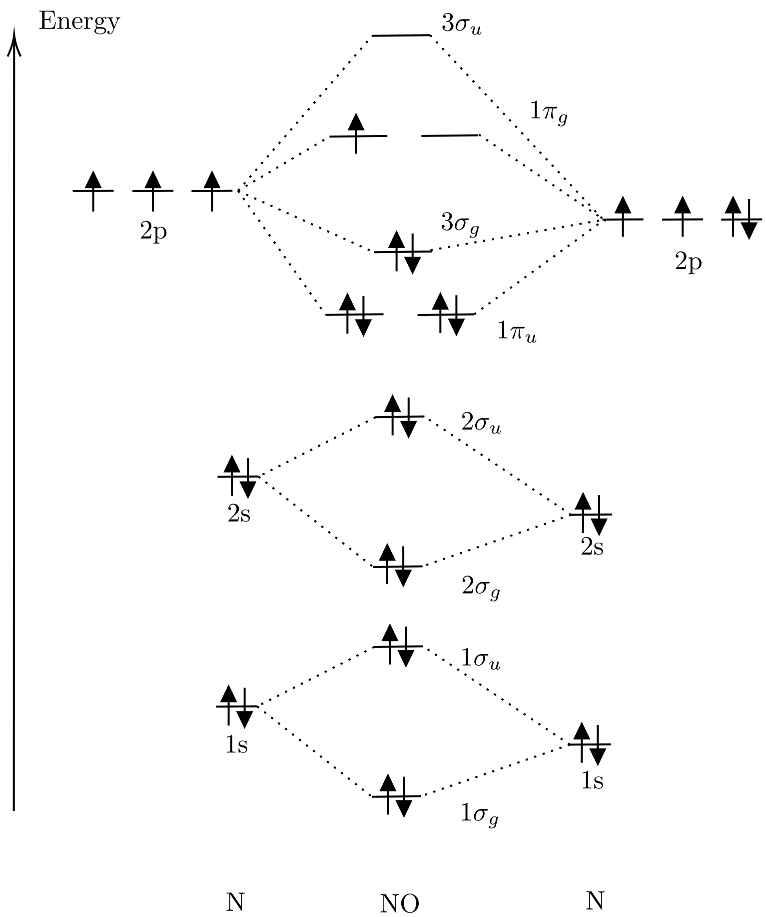
   where β is a real number and $|1\rangle$ and $|2\rangle$ are two orthogonal states, satisfying

   $$\langle 1|2\rangle = 0, \quad \langle 1|1\rangle = \langle 2|2\rangle = 1, \text{ and } |1\rangle\langle 1| + |2\rangle\langle 2| = \hat{I}. \tag{5}$$

   (a) Find the eigenvalues and eigenvectors of $\hat{H}$. Here, the eigenvectors have to be expressed as linear combinations of $|1\rangle$ and $|2\rangle$.

   (b) If the initial state of the system is $|\Psi(t = 0)\rangle = |1\rangle$, find the solution of the Schrödinger equation at time $t > 0$.

   (c) (*Optional*) Calculate $\langle\Psi(t)|\Psi(t)\rangle$ with your answer to (b), and discuss why the Hamiltonian of an isolate system must be an Hermitian operator.


Energy
$3\sigma_u$
$1\pi_g$
2p
$3\sigma_g$
2p
$1\pi_u$
$2\sigma_u$
2s
2s
$2\sigma_g$
$1\sigma_u$
1s
1s
$1\sigma_g$
N
NO
N