

**October 9, 2023**

1. Consider a one-dimensional Schrödinger equation with a time-independent potential  $V(x)$ . For a solution  $\psi(x)$  of the eigenequation  $H\psi(x) = E\psi(x)$ , show that if  $\lim_{x \rightarrow \pm\infty} \psi(x) = 0$ , the solution  $\psi(x)$  is nondegenerate and also a real function except for a possible phase factor.
2. Solve the eigenvalues and eigenfunctions for the Hamiltonian of a particle in a one-dimensional box with the potential

$$V(x) = \begin{cases} 0, & -\frac{a}{2} < x < \frac{a}{2}, \\ \infty, & \text{otherwise,} \end{cases} \quad (1)$$

where  $a > 0$  being the width of the box.