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1. The open quantum mechanics is focusing on capturing the dynamics of one system that is coupled with a thermodynamic reservoir. The total system Hamiltonian can be written as

$$H_{\mathrm{T}} = H_{\mathrm{S}} + H_{\mathrm{B}} + H_{\mathrm{SB}}, \tag{1}$$

where H_{S} is the subsystem Hamiltonian, H_{B} is the bath Hamiltonian, and H_{SB} describes the interactions between them. Here, the bath has an infinite number of degrees of freedom and arrives at the thermodynamic limit. The total system density operator satisfies the isolate system Liouville-von Neumann equation,

$$\dot{\rho}_{\mathrm{T}}(t) = -i[H_{\mathrm{T}}, \rho_{\mathrm{T}}(t)]. \tag{2}$$

Define the subsystem density operator as

$$\rho_{\mathrm{S}}(t) \equiv \mathrm{tr}_{\mathrm{B}}\rho_{\mathrm{T}}(t), \tag{3}$$

where tr_{B} denotes the partial trace over the bath degrees of freedom. Specifically, given a set of complete basis of bath, $\{|\psi_j^{\mathrm{B}}\rangle\}$, we have

$$\rho_{\mathrm{S}} = \mathrm{tr}_{\mathrm{B}}\rho_{\mathrm{T}}(t) = \sum_j \langle\psi_j^{\mathrm{B}}|\rho_{\mathrm{T}}(t)|\psi_j^{\mathrm{B}}\rangle, \tag{4}$$

Suppose the interaction has the form of

$$H_{\mathrm{SB}} = \hat{Q}_{\mathrm{S}}\hat{F}_{\mathrm{B}}, \tag{5}$$

with $\hat{Q}_{\mathrm{S}}$ being a subsystem operator and $\hat{F}_{\mathrm{B}}$ being a bath operator. Show that

$$\dot{\rho}_{\mathrm{S}}(t) = -i[H_{\mathrm{S}}, \rho_{\mathrm{S}}(t)] - i[\hat{Q}_{\mathrm{S}}, \varrho(t)], \tag{6}$$

where

$$\varrho(t) \equiv \mathrm{tr}_{\mathrm{B}}[\hat{F}_{\mathrm{B}}\rho_{\mathrm{T}}(t)]. \tag{7}$$