September 18, 2023

1. An operator $\hat{A}$, representing the observable A, has two normalized and orthogonal eigenstates $|\psi_1\rangle$ and $|\psi_2\rangle$, with the eigenvalues a_1 and a_2, respectively. Another operator $\hat{B}$, representing the observable B, also has two normalized and orthogonal eigenstates $|\phi_1\rangle$ and $|\phi_2\rangle$, with the eigenvalues b_1 and b_2. There exist relations,

$$|\psi_1\rangle = \frac{3}{5}|\phi_1\rangle + \frac{4}{5}|\phi_2\rangle \quad \text{and} \quad |\psi_2\rangle = \frac{4}{5}|\phi_1\rangle - \frac{3}{5}|\phi_2\rangle. \tag{1}$$

   (a) Observable A is measured with value a_1 obtained. What is the state of the system (immediately) after the measurement?

   (b) If B is now measured, what are the possible results and what are the corresponding probabilities?

   (c) Right after the measurement of B, A is measured again. What is the probability of obtaining a_1? (Use the law of total probability.)

2. In this problem, we present the Einstein–Podolsky–Rosen (EPR) paradox. Consider two particles with positions, x_1 and x_2, and momenta, p_1 and p_2.

   (a) Show that

$$[x_1 - x_2, p_1 + p_2] = 0. \tag{2}$$

   Then one can produce the system at state with certain $x_1 - x_2$ and $p_1 + p_2$ value. Let assume for example $x_1 - x_2 = x_0$ and $p_1 + p_2 = 0$.

   (b) Show that the measurement of x_2 would affect the momentum of the other particle, even when x_0 is large enough to make the interaction negligible.