

October 9, 2023

1. Consider a one-dimensional Schrödinger equation with a time-independent potential $V(x)$. For a solution $\psi(x)$ of the eigenequation $H\psi(x) = E\psi(x)$, show that if $\lim_{x \rightarrow \pm\infty} \psi(x) = 0$, the solution $\psi(x)$ is nondegenerate and also a real function except for a possible phase factor.
2. Solve the eigenvalues and eigenfunctions for the Hamiltonian of a particle in a one-dimensional box with the potential

$$V(x) = \begin{cases} 0, & -\frac{a}{2} < x < \frac{a}{2}, \\ \infty, & \text{otherwise,} \end{cases} \quad (1)$$

where $a > 0$ being the width of the box.