October 14, 2023

1. The time–independent Schrödinger equation of a particle is given by

$$\left[-\frac{1}{2m}\frac{d^2}{dx^2} - \frac{a^2}{m}\text{sech}^2(ax)\right]\psi(x) = \varepsilon\psi(x). \quad (\hbar \equiv 1) \tag{1}$$

(a) Show that $e^{ikx}[\tanh(ax) + c]$ can be a solution by selecting a proper c. Based on the result, calculate the reflection and transmission coefficients. (b) Check that $A\text{sech}(ax)$ is the ground state wave function and normalize it.