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1. This problem is to explain why we use the dipole–electric field interaction in the spectroscopy of molecules. The Hamiltonian for an atom in an external electromagnetic field is

$$H = \frac{1}{2m}\left[\hat{\boldsymbol{p}} - q\boldsymbol{A}(t,\hat{\boldsymbol{r}})\right]^2 + V(\hat{\boldsymbol{r}}). \tag{1}$$

Express it in the coordinate representation as

$$H = -\frac{1}{2m}\left[\nabla - iq\boldsymbol{A}(t,\boldsymbol{r})\right]^2 + V(\boldsymbol{r}). \tag{2}$$

The dipole approximation means within the atomic size the vector potential approximately remains constant with respect to $\boldsymbol{r}$, that is, $\boldsymbol{A}(t,\boldsymbol{r}) \simeq \boldsymbol{A}(t,\boldsymbol{r}_0) \equiv \boldsymbol{A}(t)$. Show that in the local gauge transformation of wave function,

$$\psi(t,\boldsymbol{r}) \to \psi'(t,\boldsymbol{r}) \equiv e^{-iq\boldsymbol{A}(t)\cdot\boldsymbol{r}}\psi(t,\boldsymbol{r}), \tag{3}$$

the Hamiltonian becomes

$$H \to H' = -\frac{\nabla^2}{2m} + V(\boldsymbol{r}) - \boldsymbol{\mu}\cdot\boldsymbol{E}(t), \tag{4}$$

with the Schrödinger equation remaining

$$i\dot{\psi}' = H'\psi'. \tag{5}$$

Here $\boldsymbol{\mu} \equiv q\boldsymbol{r}$ and $\dot{\boldsymbol{A}} = -\boldsymbol{E}$.