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1. The Hamiltonian for a free particle of mass M and spin $\hat{\boldsymbol{S}}$ placed in an external magnetic field $\boldsymbol{B}$ in the z direction is

$$H = \frac{\hat{\boldsymbol{p}}^2}{2M} - g|\boldsymbol{B}|\hat{S}_z, \tag{1}$$

   where g is a constant proportional to the magnetic moment of the particle. Give the equations that govern the time-dependence of the expectation values of all three components of $\hat{\boldsymbol{S}}$.

2. The Hamiltonian for a spin–1 particle reads

$$H = A\hat{S}_z + B\hat{S}_x^2, \tag{2}$$

   with A and B constants. Obtain the energy levels of the system. If at $t = 0$, the system is in the eigenstate of $\hat{S}_z = \hbar$, obtain the expectation value of $\hat{\boldsymbol{S}}$ as a function of time ($t > 0$).